Math 120 **Name:** ______________________________

Fall 2015

Exam #3

12/10/2015

Time Limit: 75 Minutes

You may *not* use your books or notes on this exam. You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Grade Table (for teacher use only)

Question	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total:	100	

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1. (20 points) Show using induction that $n^3 - n$ is divisible by 3 whenever n is a positive integer. (Hint: 0 is indeed divisible by 3).

First, note that $3|1^3 - 1 = 0$. Now, we assume $P(k)$ and use it to show $P(k+1)$: since

$$(k+1)^3 - (k+1) = (k^3 + 3k^2 + 3k + 1) - (k+1) = (3k^2 + 3k) + (k^3 - k)$$

and since we assume $3|k^3 - k$, the problem reduces to showing that 3 divides $(3k^2 + 3k)$. Since a 3 factors out of this expression, we are done.

2. (20 points) Prove using strong induction that every amount of postage that is at least 12 cents can be made from 4-cent and 5-cent stamps.

I basically did this in class already. It's also a pretty well used question, google it if you don't see what I did in your notes.

3. (20 points) What is the product rule (not from calculus, the product rule from this class)? Use it to show that the order of the power set $P(A)$ for any finite set A is $2^{|A|}$.

Note that for every element in the set, you can put it in one of two categories when building a subset: the element either IS in the subset, or is NOT in the subset. Thus, every element contributes 2^1 to the total number of subsets, multiplying them all together gives you $2^{|}A|$.

4. (20 points) How many subsets with more than two elements does a set of 100 elements have?

There are many approaches to this problem, one is the following: calculate the total number of subsets possible, then remove the number of subsets with 0, 1, and 2 elements. This is below:

$$2^{100} - 10^{C}0 - 10^{C}1 - 10^{C}2$$

5. (20 points) Prove using the pigeonhole principle that a map $f : A \to B$ where $|A| \neq |B|$ cannot be a bijection.

Again, did this in class. The point is ultimately that you can't have a 1-1 map from a larger set to a smaller set.