Math 142(1) **Name:** ______________________________
Spring 2015
Exam #1
3/9/2015
Time Limit: 75 Minutes

You may *not* use your books or notes on this exam. You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Grade Table (for teacher use only)

Question	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total:	100	

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1. (20 points) Find the most general antiderivative of the following functions. You need not simplify your answer where applicable.

   (i)

$$f(x) = x^{41} + x^{\frac{5}{2}}$$
$$F(x) = \frac{1}{42}x^{42} + \frac{2}{7}x^{\frac{7}{2}}$$

   (ii)

$$g(x) = \frac{x + x^2 + x^3}{\sqrt{x}}$$
$$G(x) = \frac{2}{3}x^{3/2} + \frac{2}{5}x^{5/2} + \frac{2}{7}x^{7/2}$$

   (iii)

$$h(x) = 2cos(x) + sec^2(x)$$
$$H(x) = 2sin(x) + tan(x)$$

2. (20 points) Using the limit-sum definition for area under a curve, calculate $\int_0^1 (x^2+3x+2)\ dx$.

$$\left(Hint: \sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \quad \int_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \right)$$

We have:

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} \Delta x f(x_i)$$

In our case,

$$\Delta x = \frac{b-a}{n} = \frac{1}{n}, \qquad x_i = a + i\Delta x = \frac{i}{n}$$

As such,

$$\int_0^1 \left(x^2+3x+2\right) = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left(\frac{i^2}{n^2} + 3\frac{i}{n} + 2\right) = \lim_{n\to\infty} \frac{1}{n}\left[\frac{1}{n^2}\sum_{n=1}^{n} i^2 + \frac{3}{n}\sum_{i=1}^{n} i + 2\sum_{i=1}^{n} 1\right]$$

$$= \lim_{n\to\infty} \left(\frac{1}{n}\left[\frac{n(n+1)(2n+1)}{6n^2} + \frac{3n(n+1)}{2n} + 2n\right]\right)$$

$$= \lim_{n\to\infty} \left(\frac{n(n+1)(2n+1)}{6n^3} + \frac{3n(n+1)}{2n^2} + 2\right)$$

$$= \frac{2}{6} + \frac{3}{2} + 2$$

3. (20 points) (a) (5 points) Let $F'(x) = f(x)$. From the evaluation theorem, complete the following equality:

$\int_a^b f(x)\,dx = F(b) - F(a)$

(b) (15 points) Evaluate the following integrals:

(i) $\int_{-1}^{1} t(1-t)^2\,dt = \left(\frac{1}{2}t^2 - \frac{2}{3}t^3 + \frac{1}{4}t^4\right)\Big|_{-1}^{1}$.

(ii) $\int_0^1 (x^3 + x^2)\,dx = \left(\frac{1}{4}x^4 + \frac{1}{3}x^3\right)\Big|_0^1$.

(iii) $\int_{-9}^{8} \pi\,dx = \pi(8 + 17)$

4. (20 points) (a) (5 points) Describe the difference between the following two expressions:

$$\int_2^x f(t)\,dt \qquad \& \qquad \int_2^9 f(t)\,dt$$

The first expression is a function. The second one is a constant, because it's the area under a curve.

(b) (15 points) Calculate the derivative for $f(x) = \int_2^{x^2} \left((t^2+t)\,cos(t)\right)\,dt$.

$$\frac{d}{dx}\left(\int_2^{x^2} \left(\left(t^2+t\right)cos(t)\right)\right) \stackrel{u=x^2}{=} \frac{d}{du}\left(\int_2^{u} \left(\left(t^2+t\right)cos(t)\right)\right)\frac{du}{dx}$$

$$= \left(\left(u^2+u\right)cos(u)\right)(2x) = ((x^4+x^2)(cos(x^2)))(2x)$$

5. (20 points) Calculate the following antiderivatives.

(i)

$$\int x^2\sqrt{x^3+1}\,dx$$

Let $u = x^3 + 1$, so $du = 3x^2$.

$$\int x^2\sqrt{x^3+1}\,dx = \int \frac{1}{3}\sqrt{u}\,du = \frac{1}{3}\cdot\frac{2}{3}u^{3/2} = \frac{2}{9}(x^3+1)^{3/2}$$

(ii)

$$\int x\sin(x^2)\,dx$$

Let $u = x^2, du = 2x$. Then,

$$\int xsin(x^2)dx = \int \frac{1}{2}sin(u)du = \frac{-1}{2}cos(u) = -\frac{1}{2}cos(x^2).$$

(iii)

$$\int \frac{\cos(x)}{\sin^2(x)}\,dx$$

Let $u = sin(x), du = cos(x)$.

$$\int \frac{\cos(x)}{\sin^2(x)}\,dx = \int \frac{1}{u^2}du = -u^{-1} = -(sin(x))^{-1}$$