Math 142(2) **Name:** ______________________________

Spring 2015

Exam #1

3/30/2015

Time Limit: 75 Minutes

You may *not* use your books or notes on this exam. You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Grade Table (for teacher use only)

Question	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total:	100	

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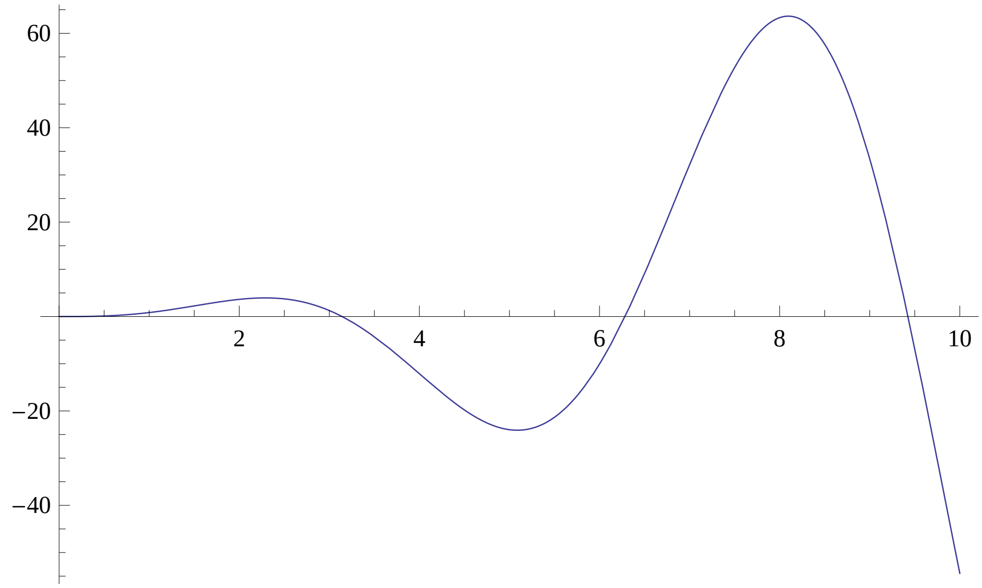
1. (20 points) Consider the following function. Is it possible to find an inverse for this function? Why or why not?



This function fails the horizontal line test; it is not 1-1. Therefore, it has no inverse.

2. Let $f(x) = 2x + cos(x)$. Find $(f^{-1})'(1)$.

First, note that $f(x)$ is both continuous and differentiable (and 1-1). This can be seen by evaluating $f'(x)$, as it is always positive. Next, note that $f^{-1}(1) = 0$, and that $f'(x) = 2 - sin(x)$. As such,

$$(f^{-6})\prime(1) = \frac{1}{2 - sin(0)} = \frac{1}{2}$$

2. (20 points) Calculate the following:

(a) (5 points) $\frac{d}{dx}\left[ln(x^3 sin^2(x))\right]$

$$\frac{1}{x^3 sin^2(x)}\left(3x^2 sin^2(x) + x^2 sin(x) cos(x)\right)$$

(b) (5 points) $\int \frac{x^4}{x^5+294}$

Let $u = x^5 + 294$, then

$$\int \frac{x^4}{x^5+294} = \frac{1}{5}\int \frac{1}{u} du = \frac{1}{5} ln|u| = \frac{1}{5} ln|x^5 + 294|$$

(c) (10 points) Using logarithmic differentiation, calculate the following derivative:

$$\frac{d}{dx}\left[\frac{x^{3/4} sin(x)\sqrt{(x+2)^3}}{(2x+1)^3\sqrt{sin(x)}}\right]$$

Calling that function y and taking the natural log of both sides, we have

$$ln(y) = \frac{3}{4} ln(x) + ln(sin(x)) + \frac{3}{2} ln(x+2) - 3ln(2x+1) - \frac{1}{2} ln(sin(x))$$

$$\frac{dy}{dx}\frac{1}{y} = \frac{3}{4}\cdot\frac{1}{x} + \frac{cos(x)}{sin(x)} + \frac{3}{2}\cdot\frac{1}{x+2} - 3\cdot\frac{2}{2x+1} - \frac{1}{2}\cdot\frac{cos(x)}{sin(x)} = \Gamma$$

Thus the answer is,

$$y' = \Gamma\cdot\left[\frac{x^{3/4} sin(x)\sqrt{(x+2)^3}}{(2x+1)^3\sqrt{sin(x)}}\right]$$

3. (20 points) Calculate the following:

(a) (5 points) $\frac{d}{dx}\left[e^{4x^3}\right]$

$$e^{4x^3} \cdot 12x^2$$

(b) (5 points)

$$\int \frac{e^{\frac{1}{x}}}{x^2}\, dx$$

Let $u = \frac{1}{x}$, $du = -\frac{1}{x^2}dx$

$$= -\int e^u du = -e^u = -e^{\frac{1}{x}}$$

(c) (5 points) $\frac{d}{dx}\left[e^{2x+x^4}\right]$

$$e^{2x+x^4} \cdot (2x + 4x^3)$$

(d) (5 points) $\frac{d}{dx}\left[4^x log_4(x^2)\right]$

Using the product rule and the change of base formula for logs, we have

$$4^x ln(4) log_4(x^2) + 4^x \frac{2x}{x^2 \cdot ln(4)}$$

4. (20 points) A bacteria culture initially contains 100 cells and grows at a rate proportional to its size. After an hour the population has increased to 420.

   1. Find an expression for the number of bacteria after t hours.
   2. Find the number of bacteria after 3 hours.
   3. When will the population reach 10,000?

   Let $B(t)$ be the bacteria after t hours. We have from the question that $\frac{dB}{dt} = kB$. As such,

   $$B(t) = B(0)e^{kt}$$

   Now first, note that $B(0) = 100$. Second, note that $B(1) = 420$. Combining these, we have

   $$B(1) = 420 = 100e^{1 \cdot k} \Rightarrow k = ln\left(\frac{42}{10}\right)$$

   As such,

   $$B(t) = 100e^{\ln\left(\frac{42}{10}\right)t}$$

   To answer $2, 3$ we simply plug in 3 for t, and for 3 we set this equation equal to 10,000 and solve for t.

5. (20 points) Calculate the following:

1. $\frac{d}{dx}\left[cos^{-1}(e^{2x})\right]$

$$\frac{-1}{\sqrt{1-[e^{2x}]^2}} \cdot e^{2x} \cdot 2$$

2. $\frac{d}{dx}\left[tan^{-1}(sin(x))\right]$

$$\frac{1}{1+sin^2(x)} \cdot cos(x)$$

3. $\frac{d}{dx}\left[sin^{-1}(ln(cos(x)))\right]$

$$\frac{1}{\sqrt{1-(ln(cos(x)))^2}} \cdot \frac{-sin(x)}{cos(x)}$$