Math 142 **Name:** ____________________
Fall 2015
Exam #3
12/10/2015
Time Limit: 75 Minutes

You may *not* use your books or notes on this exam. You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Grade Table (for teacher use only)

Question	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total:	100	

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1. (20 points) Find the area enclosed by the following curves.

$$y = 12 - x^2, \qquad y = x^2 - 6$$

Because these intersect at $x = \pm 3$, the answer is ultimately

$$\int_{-3}^{3} \left[(12 - x^2) - (x^2 - 6) \right] dx$$

2. (20 points) Find the volume of the solid obtained by rotating the region bounded by $y = 4 - x^2$, and $y = 0$ around the x-axis.

Note: there was confusion, so this is not the same as the original question.

There are two ways to do this, but I will do it in the most straightforward way: this area is bound from $x = -2$ to $x = 2$, and our volume is

$$V = \int_{-2}^{2} \pi (4 - x^2)^2 dx$$

Squaring and using the evaluation theorem completes this question.

3. (20 points) Use cylindrical shells to find the volume of the solid obtained by rotating about the y-axis the region under the curve $y = 3x - 2x^2 + 3$ from 0 to 1.

This is a straightforward application of the cylindrical shells method:

$$V = \int_0^1 2x\pi(3x - 2x^2 + 3)$$

4. (20 points) Determine the arc-ength of the function $y = cos(x)$ from $x = 0$ to $x = \pi$. Remark: it is not necessary to give a numerical answer. A correctly defined integral will suffice.

$$L = \int_0^{\pi} \sqrt{1 + (-sin(x)^2)}$$

5. (20 points) Solve the following differential equation:

$$(y^2 + xy^2)\frac{dy}{dx} = 1$$

As this is separable, proceed as in class:

$$y^2 dy = \frac{1}{1+x}dx$$

$$\Rightarrow \frac{1}{3}y^3 + C = \ln|x+1|$$