Math 115 **Name:** ____________________
Spring 2014
Exam #2
4/2/2014
Time Limit: 75 Minutes

You may *not* use your books, notes, or graphing calculator on this exam. You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Grade Table (for teacher use only)

Question	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total:	100	

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1. (20 points) (a) (10 points) Divide,

$$\frac{16x - x^3}{x^2 - x - 12} \div \frac{x^3}{2x^2 + x - 15}$$

$$\frac{16x - x^3}{x^2 - x - 12} \times \frac{2x^2 + x - 15}{x^3} = \frac{x(4+x)(4-x)}{(x-4)(x+3)} \times \frac{(x+3)(2x-5)}{x^3} = \frac{-x(4+x)(2x-5)}{x^3}$$

$$= \frac{-(4+x)(2x-5)}{x^2}$$

(b) (10 points) Solve for x:

$$\frac{5}{x-2} - \frac{3}{x^2 - 2x} = \frac{13}{4x}$$

Multiplying the leftmost term by $\frac{x}{x}$ and adding (but really, I mean subtracting), we get:

$$\frac{5x - 3}{x^2 - 2x} = \frac{13}{4x}$$

There are a number of ways to proceed from here, one of which is cross multiplication (another way is to find the least common multiple of $x^2 - 2x$ and $4x$, then multiplying both sides of the equation by this multiple). In any case, we get:

$$13x^2 - 26x = 20x^2 - 12x \Rightarrow 0 = 7x^2 + 14x \Rightarrow 0 = x(7x + 14)$$

So, $x = 0$ and $x = -2$. But, $x = 0$ makes no sense, so it is not a solution (it causes undefined terms in our equation).

2. (20 points) (a) (10 points) Solve for x:

$$a = \frac{x+b}{x+c}$$

$$ax + ac = x + b \Rightarrow ax - x = b - ac$$

Factoring,

$$x(a-1) = b - ac \Rightarrow x = \frac{b-ac}{a-1}$$

(b) (10 points) Factor completely:

$$x^3 + x^2 - 4x - 4$$

Grouping these terms together,

$$x^3 + x^2 - 4x - 4 = x(x^2-4) + x^2 - 4 \Rightarrow (x^2-4)(x+1) = (x+2)(x-2)(x+1)$$

3. (20 points) (a) (10 points) Simplify:

$$\frac{9^{-3} \cdot 3^{-4}}{3^{-2} \cdot 9^{2}}$$

Based just on rules of exponents,

$$= 9^{-5}3^{-2}$$

seeing that $9 = 3^2$,

$$= (3^2)^{-5}3^{-2} = 3^{-10}3^{-2} = 3^{-12}$$

(b) (10 points) Simplify:

$$\frac{1 - \frac{2}{x}}{1 + \frac{2}{x} - \frac{8}{x^2}}$$

The *lcm* of $1, x, x^2$ is just x^2. So, i multiply this fraction by $\frac{x^2}{x^2}$:

$$= \frac{x^2 - 2x}{x^2 + 2x - 8} = \frac{x(x-2)}{(x-2)(x+4)} = \frac{x}{x+4}$$

4. (20 points) (a) (10 points) Solve for x:

$$x^2 = -4x - 4$$

$$0 = x^2 + 4x + 4 = (x+2)(x+2)$$

hence, $x = -2$ (there is only one solution for x).

(b) (10 points) Solve for x:

$$y = \frac{2x-7}{1-5x}$$

Multiplying both sides by $1 - 5x$,

$$y - 5xy = 2x - 7 \Rightarrow y + 7 = 2x + 5xy = x(2+5y) \Rightarrow x = \frac{y+7}{2+5y}$$

5. (20 points) (a) (10 points) A math professor vacationing in Kauai jumped off a 20-foot waterfall into a pool of water below. The height of the math professor above the pool of water can be modeled by

$$H(t) = -16t^2 + 4t + 20$$

where $H(t)$ represents the height of the professor above the pool in feet t seconds after jumping off the waterfall.

1. What was the height of the professor above the pool $\frac{1}{2}$ second after jumping off the waterfall?
2. How many seconds did it take for the professor to hit the pool of water below?

The aim of this question was really to test whether or not you could analyze what a function actually means. To answer (1), we just plug in 1/2 for t to evaluate for $H(1/2)$,

$$H(\frac{1}{2}) = -16(\frac{1}{4}) + 4(\frac{1}{2}) + 20 = -4 + 2 + 20 = 18\ feet$$

In answering (2), we'd like to solve for t when $H(t) = 0$, i.e., we want to find solutions of

$$-16t^2 + 4t + 20 = 0 \Rightarrow -4t^2 + t + 5 = 0 = (-x - 1)(4x - 5)$$

And so, we have two possible solutions: $x = -1, x = \frac{5}{4}$. Now only one makes sense - $x = \frac{5}{4}$, because talking about 'negative 1 second ago' doesn't make sense.

(b) (10 points) With long division, calculate $2x^3 - 15x + 5 \div x + 3$.

$$\begin{array}{r} 2x^2 \quad - 6x + 3 \\ x + 3 \overline{\big) \; 2x^3 \qquad\quad - 15x + 5} \\ \underline{-\, 2x^3 - 6x^2 \qquad\qquad} \\ - 6x^2 - 15x \qquad \\ \underline{6x^2 + 18x} \qquad \\ 3x + 5 \\ \underline{- 3x - 9} \\ - 4 \end{array}$$