* Proceedings of the 1983 Beijing symposium on differential geometry and differential equations. 87m:58007
Held in Beijing, August 29–September 10, 1983.
Edited by Shan Tao Liao and S. S. Chern.
— Science Press, Beijing, 1986. viii+804 pp.

RESTRICTED PROBLEM OF MOTION OF THREE VORTICES AND RELATED TOPICS

Qian Min (钱敏) and Jiang Yunping (蒋云平)
(Department of Mathematics, Peking University)

The motion of three vortices with strength k_1, k_2, k_3 is considered. The case for $k_3=0$ physically signifies that we neglect the reaction of the third vortex on the other two. This case is analysed in detail in this paper. Since for $k_3=0$, the equations of motion of the other two vortices can be integrated and are found to be stationary in a rotating frame of reference, we first consider the system in this coordinate system. By eliminating singular points (where the differential equation is undefined) on the boundary of the physical region, we obtain a regular planar system which is equivalent to the original one in the physical region outside of which the problem is of no interest whatever. The equivalent regular system can be analysed by using phase portrait techniqes, so that the behavior of the solution can be fully understood. By transforming back to the original coordinate system, we get the following main theorem:

The equation of motion of a vortex system with three vortices, one of which has strength zero is

$$\dot{x}_1 = -\frac{k_2}{4\pi}(Y_1-Y_2)/\,l_{12}^2\,,$$

$$\dot{x}_2 = \frac{k_1}{4\pi}(Y_1-Y_2)/\,l_{12}^2\,,$$

$$\dot{Y}_1 = \frac{k_2}{4\pi}(x_1-x_2)/\,l_{12}^2\,.$$

$$\dot{Y}_2 = -\frac{k_1}{4\pi}(x_1 - x_2)/\, l_{12}^2 \,,$$

$$\dot{x}_3 = \frac{k_1}{4\pi}\,\frac{Y_1 - Y_3}{l_{13}^2} + \frac{k_2}{4\pi}\,\frac{Y_2 - Y_3}{l_{23}^2} \,,$$

$$\dot{Y}_3 = -\frac{k_1}{4\pi}\,\frac{x_1 - x_2}{l_{13}^2} - \frac{k_2}{4\pi}\,\frac{x_2 - x_3}{l_{23}^2}$$

((x_i, Y_i) are coordinates of the ith vortex, l_{ij} is the distance between the i and the jth vortex).

<u>Theorem</u>. If $k_1 \cdot k_2 < 0$, $k_1 + k_2 \neq 0$, then system (I) has a unique stable periodic solution. If $k_1 \cdot k_2 > 0$, the system has infinite many periodic solutions and infinite many quasiperiodic solutions.

References

[1] Chorin A.J. & Marsden, J.E. A Mathematical Introduction to Fluid Mechanics, Springer, 1979.

[2] Novikov, E.A. Dynamics and statics of vortex systems J. Exp. Theor. Phys., 68 (1975), 5, 1868-1882.

[3] Aref H. Motion of three vortices, Phys. Fluids 22(1979), No. 3, March, 393-400.

[4] Zhang Jin Yan, Geometrical Theory of O.D.E. and Bifurcation. Peking University Press, 1991.