

**Renormalization and Geometry in
One-Dimensional and Complex Dynamics**

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One-Dimensional and Complex Dynamics**

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To my Mother and Father

To Bin and Jeffrey

Preface

This monograph summarizes my research in dynamical systems during the past eight years. Included too are many facts, techniques, and results which I have learned from others and which have greatly enhanced my own work.

In September 1985, I arrived in the United States to pursue a doctoral degree in mathematics. One and a half years later, I wrote to Dennis Sullivan and asked if I could study under his supervision. I became a Ph.D. student of Sullivan and so began this research. During my years as his student, Sullivan gave classes and ran seminars on Tuesday and Thursday at the Graduate Center of the City University of New York, where he held an Einstein Chair. These sessions sometimes lasted all day.

My first problem in this area of research was suggested by Sullivan in one of his classes. The problem was first, to understand the asymptotic geometry of Cantor sets generated by a family of dynamical systems involving a singular point and second, to investigate the geometric property of conjugacy between two such dynamical systems. This work is included in Chapter Two of the present monograph as an application of the Koebe distortion principle. One recognized program is to “fill in” the dictionary between the theory of one-dimensional dynamical systems and the theory of Kleinian groups. Seeking to advance this program and to generalize my first research in this area, I studied the space of geometrically finite one-dimensional maps and the classification of these maps up to conjugacy by quasimetric homeomorphisms and up to conjugacy by diffeomorphisms. This work is described in Chapter Three. Part of it comes from my Ph.D. thesis. Frederick Gardiner, Charles Tresser, and Richard Sacksteder gave me their help during this study and Sullivan provided his own insightful suggestions.

In 1987, Peter Veerman gave me a paper of Robert MacKay concerning Denjoy’s theorem for circle diffeomorphisms. In this paper, MacKay applies

the renormalization method to an old theorem. I became interested in this approach. It is described in Chapter One as an introduction to renormalization theory and as an application of the Denjoy distortion principle. That same year, Welington de Melo and Sebastian van Strien visited the Einstein Chair and presented their results on one-dimensional dynamical systems. Grzegorz Świątek also paid a visit and presented his results on critical circle mappings. Interestingly, they had independently developed a technique to estimate the distortion of a one-dimensional map having a critical point. This technique was later generalized to a larger class of one-dimensional maps by Sullivan; it is called the Koebe distortion principle because of its similarity to Koebe's distortion theorem in one complex variable discovered some eighty years ago. Chapter Two contains several versions of the distortion principle.

A universal rule governs the transition from simple motion to chaos in a one-parameter family of dynamical systems with a unique quadratic critical point. Mitchell Feigenbaum discovered this in the 1970s. The rule can be explained by means of a family of one-dimensional dynamical systems like those generated by quadratic polynomials. Feigenbaum calculated period doubling bifurcations for such a family and showed that the limit of these period doubling bifurcations is a chaotic dynamical system in the family. Furthermore, the appearance of the chaotic dynamical system follows a universal pattern which is described by the so-called Feigenbaum universal number. Oscar Lanford III gave the first proof of this discovery with some computer help. For the chaotic dynamical system, the interesting object is its attractor. The attractor is uncountable, perfect, and totally disconnected: a Cantor set. Feigenbaum, and independently, Pierre Couillet and Charles Tresser, discovered in the 1970s that the geometry of this Cantor set is universal, meaning that it does not depend on the specific family being studied. This discovery is similar to Mostow's rigidity theorem, which says that in the class of closed hyperbolic three-manifolds, topology determines geometry. During my years as a Ph.D. student, some work of Sullivan led to an important mathematical understanding of this discovery. Chapter Four contains part of this work based on my class notes.

During my time as his Ph.D. student, Sullivan showed me how to deform a Feigenbaum-like map. I began to think about this topic and also to study the spectrum of the period doubling operator. Meanwhile, Takehiko Morita visited the Einstein Chair. I told him what I was working on and he showed me a general strategy to study the spectrum of a transfer operator in thermodynamical formalism. I applied this strategy to the study of the tangent map of the period doubling operator by connecting it with a transfer

operator. This led eventually to a conceptual proof of the existence of the Feigenbaum universal number in a joint paper with Morita and Sullivan. This is the origin of Chapter Six. During this study, conversations with David Ruelle and Henri Epstein helped me to better understand the spectrum of the period doubling operator and other related topics. In the summer of 1993, Viviane Baladi lectured on thermodynamical formalism at a workshop held in Hillerød, Denmark. After her lectures, I asked her about generalizing some of the results in her lectures to the Zygmund continuous vector space. She showed me some calculations that suggested this possibility. In the summer of 1994, I visited the Forschungsinstitut für Mathematik at the Eidgenössische Technische Hochschule in Zürich to work with Baladi and Lanford on this problem. The fruit of this work is described in a paper written by Baladi, Lanford, and myself. A special case of our result is presented in Chapter Six for the purpose of studying the spectrum of the renormalization operator.

After completing my Ph.D. study in May 1990, I went to the Institute for Mathematical Sciences at Stony Brook in September 1990. There I was influenced by John Milnor's investigations in complex dynamics; I began to work on some problems in this field.

Yakov Sinai constructed Markov partitions for Anosov dynamical systems in the 1960s. Rufus Bowen generalized this method to Axiom A dynamical systems. This method became very important in the study of hyperbolic dynamical systems. During the academic year of 1991, Mitsuhiro Shishikura visited the Institute for Mathematical Sciences at Stony Brook and presented his work in complex dynamics. In his lectures, he introduced me to the result of Jean-Christophe Yoccoz on the local connectivity of the Julia set of a non-renormalizable quadratic polynomial and to the technique called Yoccoz puzzles. This technique was first used by Bodil Branner and John Hubbard in their study of certain cubic polynomials and was successfully used by Yoccoz in his study of non-renormalizable quadratic polynomials. The technique is a little different from that of the Markov partitions but is motivated by the same philosophy. By learning this technique, I was able to apply it, along with my knowledge of infinitely renormalizable maps, to the study of infinitely renormalizable quadratic polynomials. I proved that some conditions on an infinitely renormalizable quadratic polynomial are sufficient to ensure that its Julia set is locally connected. This is described in Chapter Five.

The first time I applied Yoccoz puzzles in my research was in the study of bounded and bounded nearby geometry of certain infinitely renormalizable folding maps and in the quasisymmetric classification of these maps. In this study, I combined the technique of Yoccoz puzzles with Markov partitions and

with my previous work on geometrically finite one-dimensional maps. This research is described in Chapter Four.

After completing my contract with the State University of New York at Stony Brook, I started work, in September 1992, at Queens College of the City University of New York. I began to regularly attend Sullivan's seminars at the Graduate Center of the City University of New York. Jun Hu told me there that Sullivan had completed his work about the *a priori* complex bounds for the Feigenbaum quadratic polynomial. This caught my attention because the *a priori* complex bounds is a sufficient condition that the Julia set of a real infinitely renormalizable quadratic polynomial is locally connected. After Hu explained to me the idea of Sullivan's proof of the *a priori* complex bounds for the Feigenbaum quadratic polynomial, I went on to write a joint paper with him about the local connectivity of the Julia set of the Feigenbaum quadratic polynomial. Later I realized that unless I had a complete proof of Sullivan's result, my understanding of the local connectivity of the Julia set of the Feigenbaum polynomial was incomplete. I began a serious study of Sullivan's result, which appears in Chapter Five. During this study, several conversations with Sullivan, along with the thesis of Edson de Faria, provided a lot help. During my research into infinitely renormalizable quadratic polynomials, communication with Curt McMullen via e-mail was very helpful. Several statements were made more precise because of his comments. Moreover, they led me to combine several of my papers in this direction into one self-contained paper, which is the origin of Chapter Five.

After I explained to Sullivan my research about the local connectivity of the Mandelbrot set at certain infinitely renormalizable points, he suggested that I might be able to reduce the computational aspect of many proofs by addressing the argument from a topological perspective. This is also presented in Chapter Five. In this study, many conversations with Tan Lei helped me to better understand the topological structure of the Mandelbrot set.

The survey articles written by E. B. Vul, Ya. G. Sinai, and K. M. Khanin [VSK], by J. Milnor [MI2,MI3], and by J. Hubbard [HUB], the book written by P. Collet and J.-P. Eckmann [COE] and the book edited by P. Cvitanović [CVI] provided valuable guidance, not only at the beginning, but also throughout the whole period of my research.

An invitation from the Advanced Series in Nonlinear Dynamics gave me a chance to work on this research monograph. Its editor, Robert MacKay, not only suggested this monograph but also encouraged me to complete it. Frank Isaacs gave me advice on English and on mathematical presentation.

In addition to the names of colleagues mentioned above, Benjamin Biele-

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Queens, New York

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Bibliography

- [AH1] L. V. Ahlfors, *Lectures on Quasiconformal Mappings*. D. Van Nostrand-Reinhold Company, Inc., Princeton, New Jersey, 1966.
- [AH2] L. V. Ahlfors, *Conformal Invariants: Topics in Geometric Function Theory*. McGraw-Hill Book Co., New York, 1973.
- [AH3] L. V. Ahlfors, *Complex Analysis*. McGraw-Hill Book Co., New York, 1979.
- [ARN] V. I. Arnold, *Ordinary Differential Equations*. M.I.T. Press: Cambridge, MA., 1973. (Russian original, Moscow, 1971).
- [BAL] V. Baladi, *Dynamical zeta functions*. In *Real and Complex Dynamical Systems* (B. Branner and P. Hjorth, eds). Kluwer Academic Publishers, 1995.
- [BJL] V. Baladi, Y. Jiang, and O. E. Lanford III, *Transfer operators acting on Zygmund functions*. FIM/ETH, Zürich Preprint, March 1995 and Trans. of the Amer. Math. Soc., to appear.
- [BEF] T. Bedford and A. Fisher, *Ratio geometry, rigidity and the scenery process for hyperbolic Cantor sets*. IMS preprint 1994/9, SUNY at Stony Brook.
- [BIE] L. Bieberbach, *Conformal Mapping*. Chelsea Publishing Company, New York, 1953.
- [BSTV] B. Bielefeld, S. Sutherland, F. Tangerman, and J. J. P. Veerman, *Dynamics of certain non-conformal degree two maps of the plane*. Experimental Mathematics, **2**, 1993, pp. 281-300.
- [BLA] P. Blanchard, *Complex analytic dynamics on the Riemann sphere*. Bull. of the Amer. Math. Soc. **11**, 1984, pp. 85-141.
- [BO1] R. Bowen, *Equilibrium States and the Ergodic Theory of Anosov Diffeomorphisms*. Lecture Notes in Mathematics, Vol. **470**, Springer-Verlag, New York, Berlin, 1975.

- [BO2] R. Bowen, *Hausdorff dimension of quasi-circles*. Publ. Math. Inst. des Hautes Études Scientif., No. **50**, 1979, pp. 11-25.
- [BO3] R. Bowen, *A horseshoe with positive measure*. Invent. Math., **29** (1975), 203-204.
- [BO4] R. Bowen, *Markov partitions for Axiom A diffeomorphisms*. Amer. Journal of Math. **92**, 1970, pp. 907-918.
- [BRA] L. de Branges, *A proof of the Bieberbach conjecture*. Acta Mathematica, **154**, 1985, pp. 137-152.
- [BRH] B. Branner and J. Hubbard, *The iteration of cubic polynomials, Part I : The global topology of parameter space & Part II : Patterns and parapatterns*. Acta Math **160**, 1988, pp. 143-206 & Acta Math, 169, 1992, pp. 229-325.
- [CAR] C. Carathéodory, *Über die Begrenzung einfach zusammenhängender Gebiete*. Math. Ann. **73**, 1913, pp. 323-370. (Gesam. Math. Schr., v. 4).
- [CAG] L. Carleson and T. Gamelin, *Complex Dynamics*. Springer-Verlag, Berlin, Heidelberg, 1993.
- [CAW] E. Cawley, *The Teichmüller space of an Anosov diffeomorphism of T^2* . IMS preprint 1991/9, SUNY at Stony Brook.
- [CAE] M. Campanino and H. Epstein, *On the existence of Feigenbaum's fixed point*. Commun. Math. Phys., **79**, 1981, pp. 261-302.
- [CER] M. Campanino, H. Epstein, and D. Ruelle, *On Feigenbaum's functional equation $g \circ g(\lambda x) + \lambda g(x) = 0$* . Topology, **21**, 1982, pp. 125-129.
- [CCR] F. Christiansen, P. Cvitanović, and H. Rugh, *The spectrum of the period doubling operator in terms of cycles*. J. Phys. A, **23**, 1990, pp. L713-L717.
- [COE] P. Collet and J.-P. Eckmann, *Iterated Maps on the Interval as Dynamical Systems*. Progress in Physics, **Vol. 1**, Birkhäuser, Boston, 1980.
- [COT] P. Coullet and C. Tresser, *Itération d'endomorphismes et groupe de renormalisation*. C. R. Acad. Sci. Paris Ser., A-B **287**, 1978, pp. A577-A580. (J. Phys. Coll. 39:C5 (1978), 25-28; supplément au 39:8).
- [CUI] G. Cui, *Circle expanding maps and symmetric structures*. Preprint.
- [CVI] P. Cvitanović, *Universality in Chaos*. Adam Hilger Ltd., Bristol, 1984.
- [DEN] A. Denjoy, *Sur les courbes définies par les équations différentielles à la surface du tore*. J. Math. Pure et Appl. **11** (IV), 1932, pp. 333-375.
- [DH1] A. Douady and J. H. Hubbard, *Itération des polynômes quadratiques complexes*. C.R. Acad. Sci. Paris, **294**, 1982, pp. 123-126.
- [DH2] A. Douady and J. H. Hubbard, *Étude dynamique des polynômes complexes I & II*. Publ. Math. d'Orsay, 1984 & 1985.

- [DH3] A. Douady and J. H. Hubbard, *On the dynamics of polynomial-like mappings*. Ann. Sci. Éc. Norm. Sup., Paris, **18**, 1985, pp. 287-343.
- [DH4] A. Douady and J. H. Hubbard, *A proof of Thurston's topological characterization of rational maps*. Preprint, Institute Mittag-Leffler 1984.
- [DUS] N. Dunford and J. Schwartz, *Linear Operators*. Vol. I, II, and III, Interscience, 1985.
- [EE1] J.-P. Eckmann and H. Epstein, *Scaling of Mandelbrot sets generated by critical point preperiodicity*. Commun. Math. Phys., **101**, 1985, pp. 283-289.
- [EE2] J.-P. Eckmann and H. Epstein, *Bounds on the unstable eigenvalue for period doubling*. Commun. Math. Phys., **128**, 1990, pp. 427-435.
- [ECW] J.-P. Eckmann and P. Wittwer, *A complete proof of Feigenbaum conjectures*. J. of Stat. Phys., **46**, 1987, pp. 455-475.
- [EPS] H. Epstein, *New proofs of the existence of the Feigenbaum functions*. Commun. Math. Phys., **106**, 1986, pp. 395-426.
- [FAL] K. J. Falconer, *The Geometry of Fractal Sets*. Cambridge Univ. Press, 1985.
- [FAR] E. de Faria, *Proof of universality for critical circle mappings*, Thesis, Graduate Center of CUNY, 1992.
- [FAM] E. de Faria and W. de Melo, *Proof of universality for critical circle mappings*. Preprint in preparation.
- [FE1] M. Feigenbaum, *Quantitative universality for a class of non-linear transformations*. J. Stat. Phys., **19**, 1978, pp. 25-52.
- [FE2] M. Feigenbaum, *The universal metric properties of non-linear transformations*. J. Stat. Phys., **21**, 1979, pp. 669-706.
- [FLE] M. Flexor, *Théorème du secteur d'après D. Sullivan*, Preprint, 92-26, Université de Paris-Sud, Orsay, 1993.
- [GAR] F. Gardiner, *Teichmüller Theory and Quadratic Differentials*. A Wiley-Interscience Publication, John Wiley & Sons, New York, 1987.
- [GS1] F. Gardiner and D. Sullivan, *Lacunary series as quadratic differentials in conformal dynamics*. Contemporary Mathematics, **169**, 1994, pp. 307-330.
- [GS2] F. Gardiner and D. Sullivan, *Symmetric and quasisymmetric structures on a closed curve*. Amer. J. of Math., 114, no. 4, (1992), pp. 683-736.
- [GRS] J. Graczyk and G. Świątek, *Private talk*.
- [GU1] J. Guckenheimer, *Limit sets of S-unimodal maps with zero entropy*. Commun. Math. Phys., **110**, 1987, pp. 655-659.
- [GU2] J. Guckenheimer, *Sensitive dependence on initial conditions for one-dimensional maps*. Commun. Math. Phys., **70**, 1979, pp. 133-160.

- [GUJ] J. Guckenheimer and S. Johnson, *Distortion of S-unimodal maps*. *Annals. Math.*, **132**, 1990, pp. 71-130.
- [HER] M. R. Herman, *Sur la conjugaison différentiable des difféomorphismes du cercle á des rotations*. *Publ. Math. Inst. des Hautes Études Scientif.*, No. **49**, 1979, pp. 5-234.
- [HPS] M. W. Hirsh, and C. C. Pugh, and M. Shub, *Invariant Manifolds*. Springer Lectures Notes in Mathematics, Vol. **583**, Springer-Verlag: New York, Heidelberg, Berlin, 1977.
- [HUS] J. Hu and D. Sullivan, *Topological conjugacy of circle diffeomorphisms*. IMS preprint 1995/7, SUNY at Stony Brook.
- [HUB] J. H. Hubbard, *Local connectivity of Julia sets and bifurcation loci: three theorems of J. -C. Yoccoz*. *Topological Methods in Modern Mathematics, A Symposium in Honor of John Milnor's Sixtieth Birthday*, Publish or Perish, Inc., 1993, pp. 467-512.
- [JAK] M. Jakobson, *Quasisymmetric conjugacy for some one-dimensional maps inducing expansion*. Preprint.
- [JI1] Y. Jiang, *Geometry of Cantor systems*. Preprint. (Also *Leading gap determines the geometry of Cantor sets*. Preprint of IHES, June/1989.)
- [JI2] Y. Jiang, *Local normalization of one-dimensional maps*. Preprint of IHES, June/1989.
- [JI3] Y. Jiang, *Generalized Ulam-von Neumann transformations*. Thesis, 1990, Graduate School of CUNY and UMI dissertation service.
- [JI4] Y. Jiang, *Dynamics of certain one-dimensional mappings: I. $C^{1+\alpha}$ -Denjoy-Koebe distortion lemma*. IMS preprint 1991/1, SUNY at Stony Brook.
- [JI5] Y. Jiang, *On quasisymmetrical classification of infinitely renormalizable maps – I. Maps with Feigenbaum topology, and II. Remarks on maps with a bounded type topology*. IMS preprint 1991/19, SUNY at Stony Brook.
- [JMS] Y. Jiang, T. Morita, and D. Sullivan, *Expanding direction of the period doubling operator*. *Commun. Math. Phys.*, **144**, 1992, pp. 509-520.
- [JI6] Y. Jiang, *Asymptotic differentiable structure on Cantor set*. *Commun. Math. Phys.*, **155**, 1993, pp. 503-509.
- [JI7] Y. Jiang, *Geometry of geometrically finite one-dimensional maps*. *Commun. Math. Phys.*, **156**, 1993, pp. 639-647.
- [JI8] Y. Jiang, *Dynamics of certain non-conformal semi-groups*. *Complex variables*, Vol. **22**, 1993, pp. 27-34. (also [JI8ims] IMS preprint 1992/5, SUNY at Stony Brook under the same title.)
- [JI9] Y. Jiang, *On Ulam-von Neumann transformations*. *Commun. Math. Phys.*, **172**, 1995, pp. 449-459.

- [JI10] Y. Jiang, *Markov partitions and Feigenbaum-like maps*. Commun. in Math. Phys., **171**, 1995, pp. 351-363.
- [JI11] Y. Jiang, *Smooth classification of geometrically finite one-dimensional maps*. Trans. of the Amer. Math. Soc., 1996, to appear.
- [JI12] Y. Jiang, *On rigidity of one-dimensional maps*. Preprint.
- [JI13] Y. Jiang, *Infinitely renormalizable quadratic Julia sets*. Preprint.
- [JIH] Y. Jiang and J. Hu, *The Julia set of the Feigenbaum polynomial is locally connected*. Preprint.
- [JI14] Y. Jiang, *On Sullivan's sector theorem*. Preprint.
- [JI15] Y. Jiang, *Renormalization on one-dimensional folding maps*. Proceedings of the International Conference on Dynamical Systems and Chaos, Volume **1**, World Scientific Publishing Co. Pte. Ltd., 1995, pp. 116-125.
- [JI16] Y. Jiang, *The Renormalization method and quadratic-like maps*. MSRI preprint No. 081-95.
- [JI17] Y. Jiang, *Local connectivity of the Mandelbrot set at certain infinitely renormalizable points*. MSRI preprint No. 063-95, Berkeley.
- [KAH] J. Kahn, *Unpublished work*.
- [KAO] Y. Katznelson and D. Ornstein, *The differentiability of conjugation of certain diffeomorphisms of the circle*. Ergod. Th. & Dynam. Sys. **9**, 1989, pp. 643-680.
- [KHS] K. M. Khanin and Ya. G. Sinai, *A new proof of M. Herman's theorem*. Commun. in Math. Phys., **112**, 1987, pp. 89-101.
- [LA1] O. E. Lanford III, *A computer-assistant proof of the Feigenbaum conjecture*. Bull. of the Amer. Math. Soc., **6**, 1982, pp. 427-434.
- [LA2] O. E. Lanford III, *A shorter proof of the existence of Feigenbaum fixed point*. Commun. in Math. Phys., **96**, 1984, pp. 521-538.
- [LEV] G. Levin and S. van Strien, *Local connectivity of the Julia sets of real polynomials*. IMS preprint 1995/5.
- [LEH] O. Lehto, *Univalent Functions and Teichmüller Spaces*. Springer-Verlag, New York, Berlin, 1987.
- [LMM] R. de la Llave, J. M. Marco and R. Moriyon, *Canonical perturbation theory of Anosov systems and regularity results for the Livsic cohomology equation*. Annals of Mathematics, **123**, (1986), pp. 537-611.
- [LL1] R. de la Llave, *Invariants for smooth conjugacy of hyperbolic dynamical systems I*. Commun. Math. Phys. **109**, (1987), pp. 681-689.
- [LL2] R. de la Llave, *Invariants for smooth conjugacy of hyperbolic dynamical systems II*. Commun. Math. Phys. **109**, (1987), pp. 369-378.
- [LYM] M. Lyubich and J. Milnor, *The Fibonacci unimodal map*. J. Amer. Math. Soc. **6**(1993), pp. 425-457.

- [LYU] M. Lyubich, *Geometry of quadratic polynomials: moduli, rigidity, and local connectivity*. IMS preprint 1993/9, SUNY at Stony Brook. And later developments.
- [LYY] M. Lyubich and M. Y. Yampolsky, *Dynamics of quadratic polynomials: complex bounds for real maps*. MSRI preprint No. 034-95.
- [MAC] R. MacKay, *A simple proof of Denjoy's theorem*. Math. Proc. Camb. Phil. Soc., **103**, 1988, pp. 299-303.
- [MAM] J. M. Marco and R. Moriyon, *Invariants for smooth conjugacy of hyperbolic dynamical systems III*. Commun. Math. Phys. **112**, (1987), pp. 317-333.
- [MA1] D. Mayer, *The Ruelle-Araki Transfer Operator in Classical Statistical Mechanics*. Lecture Notes in Physics, vol. **123**, Berlin, Heidelberg, New York: Springer, 1980.
- [MA2] D. Mayer, *On the thermodynamics formalism for the Gauss map*. Commun. Math. Phys. **130**, 1990, pp. 311-333.
- [MA3] D. Mayer, *The thermodynamic formalism approach to Selberg's zeta function for $PSL(2, \mathbf{Z})$* . Bull. of the Amer. Math. Soc., Vol. **25**, No. 1, 1991, pp. 55-60.
- [MC1] C. McMullen, *Complex Dynamics and Renormalization*. Ann. of Math. Stud., vol **135**, Princeton Univ. Press, Princeton, NJ, 1994.
- [MC2] C. McMullen, *Renormalization and 3-manifolds which fiber over the circle*. Preprint.
- [MV1] W. de Melo and S. van Strien, *A structure theorem in one-dimensional dynamics*. Annals of Math., **129**, 1989, pp. 519-546.
- [MV2] W. de Melo and S. van Strien, *One-Dimensional Dynamics*. Springer-Verlag, Berlin, Heidelberg, 1993.
- [MI1] J. Milnor, *On the concept of attractor*. Commun. in Math. Phys., **99**, 1985, pp. 177-195.
- [MI2] J. Milnor, *Dynamics in one complex variable: Introductory lectures*. IMS preprint 1990/5, SUNY at Stony Brook.
- [MI3] J. Milnor, *Local connectivity of Julia sets: expository lectures*. IMS preprint 1992/11, SUNY at Stony Brook.
- [MIT] J. Milnor and W. Thurston, *On iterated maps of the interval: I and II*. In Lecture Notes in Mathematics, **Vol. 1342**, pp. 465-563, Springer, New York, Berlin, 1988, pp. 465-563.
- [MOS] D. Mostow, *Strong Rigidity of Locally Symmetric Spaces*. Ann. of Math. Stud., vol **78**, Princeton Univ. Press, Princeton, NJ, 1972.
- [NEW] S. E. Newhouse, *Nondensity of Axiom A (a) on S^2* . Proc. Symp. Pure Math., **14**, 1970, pp. 191-202.

- [NUS] R. D. Nussbaum, *The radius of the essential spectrum*. Duke Math. J. vol. **37**, 1970, pp. 473–478.
- [PAL] W. Paluba, *Talks in Seminars and Thesis*, 1992, the Graduate school of CUNY.
- [PAP] W. Parry and M. Pollicott, *Zeta Functions and the Periodic Orbit Structure of Hyperbolic Dynamics*. Société Mathématique de France, Astérisque, vol. **187-188**, Paris, 1990.
- [PAM] J. Palis and W. de Melo, *Geometric Theory of Dynamical Systems: An Introduction*. Springer-Verlag: New York, Heidelberg, Berlin, 1982.
- [PER] R. Perez-Marco, Private conversation with a third party.
- [PET] C. Petersen, *Local connectivity of some Julia sets containing a circle with an irrational rotation*. IHES preprint, April, IHES/M/1994/26.
- [PIS] T. Pignataro and D. Sullivan, *Ground state and lowest eigenvalue of the Laplacian for non-compact hyperbolic surfaces*. Commun. Math. Phys., **104**, pp. 529–535.
- [PIR] A. Pinto and D. Rand, *Global phase space universality, smooth conjugacies and renormalization: 2. The $C^{k+\alpha}$ case using rapid convergence of Markov families*. Nonlinearity, **4**, (1992), pp. 49-79.
- [PNS] A. Pinto and D. Sullivan, *The circle and solenoid*. Preprint.
- [PO1] M. Pollicott, *A complex Ruelle-Perron-Frobenius theorem and two counterexamples*. Ergod. Th. and Dynam. Sys. **4**, 1984, pp. 135-146.
- [PO2] M. Pollicott, *A note on the Artuso-Aurell-Cvitanović approach to the Feigenbaum tangent operator*. J. Stat. Phys., **60**, 1991, pp. 257-267.
- [PRZ] F. Przytycki, *On U -stability and structural stability of endomorphisms satisfying Axiom A*. Studia-Math., **60**, no. 1, 1977, pp. 61–77.
- [RAN] D. Rand, *Global phase space universality, smooth conjugacies and renormalization: 1. The $C^{1+\alpha}$ case*. Nonlinearity, **1**, (1988), pp. 181-202.
- [REI] M. Reimann, *Ordinary differential equations and quasiconformal mappings*. Invent. Math., vol. **33**, 1976, pp. 247–270.
- [RIC] S. Rickman, *Removability theorem for quasiconformal mappings*. Ann. Ac. Scient. Fenn, **499**, 1969, pp. 1-8.
- [RU1] D. Ruelle, *Statistical mechanics of a one-dimensional lattice gas*. Commun. in Math. Phys., **9**, 1968, pp. 267-278.
- [RU2] D. Ruelle, *A measure associated with Axiom A attractors*. Amer. J. Math., **98**, 1976, pp. 619-654.
- [RU3] D. Ruelle, *Thermodynamic Formalism : The Mathematical Structures of Classical Equilibrium Statistical Mechanics*. **5**, Addison-Wesley: Reading, Mass., 1978.

- [RU4] D. Ruelle, *Repellers for real analytic maps*. Ergod. Th. & Dynam. Sys., **2**, 1982, pp. 99-107.
- [RU5] D. Ruelle, *The thermodynamical formalism for expanding maps*. Commun. Math. Phys. **125**, 1989, pp. 239-262.
- [RU6] D. Ruelle, *Dynamical Zeta Functions for Piecewise Monotone Maps of the Interval*. CRM monograph series, Volume **4**, AMS, Providence, RI, 1994.
- [RUG] H. H. Rugh, *Generalized Fredholm determinants and Selberg zeta functions for Axiom A dynamical systems*. Preprint.
- [SHI] M. Shishikura, *The Hausdorff dimension of the boundary of the Mandelbrot set and Julia sets*. IMS preprint 1991/7, SUNY at Stony Brook
- [SHU] M. Shub, *Endomorphisms of compact differentiable manifolds*. Amer. J. Math., **91**, 1969, pp. 129-155.
- [SHS] M. Shub and D. Sullivan, *Expanding endomorphisms of the circle revisited*. Ergod. Th & Dynam. Sys., **5**, 1985, pp. 285-289.
- [SI1] Ya. G. Sinai, *Markov partitions and C-diffeomorphisms*. Func. Anal. and its Appl., **2**, no. 1, 1968, pp. 64-89.
- [SI2] Ya. G. Sinai, *Gibbs measures in ergodic theory*. Russian Math. Surveys, **27**, no. 4 1972, pp. 21-69.
- [SIN] D. Singer, *Stable orbits and bifurcations of maps of the interval*. SIAM J. - Appl. Math., **35**, 1978, pp. 260-267.
- [SOR] D. E. K. Sørensen, *Complex Dynamical Systems: Rays and non-local connectivity*. Ph. D. Thesis, Technical University of Denmark, 1995.
- [SW1] G. Świątek, *Rational rotation numbers for maps of the circle*. Commun. in Math. Phys., **119**, 1988, pp. 109-128.
- [SW2] G. Świątek, *Hyperbolic is dense in the real quadratic polynomials*. IMS preprint 1992/10, SUNY at Stony Brook. And later developments with J. Graczyk.
- [STA] M. Stark, *Smooth conjugacy and renormalization for diffeomorphisms of the circle*. Nonlinearity, **1**, 1988, pp. 541-575.
- [STR] S. van Strien, *On the bifurcations creating horseshoes in Dynamical Systems and Turbulence*. Lecture Notes in Mathematics, **898**, 1981, Springer, Berlin, New York, pp. 316-351.
- [SU1] D. Sullivan, *Seminar on conformal and hyperbolic geometry*. IHES Preprint, March/1982.
- [SU2] D. Sullivan, *Quasiconformal homeomorphisms and dynamics I, solution of the Fatou-Julia problem on wandering domains*. Ann. Math., **122**, 1985, pp. 401-418.

- [SU3] D. Sullivan, *Differentiable structure on fractal like sets determined by intrinsic scaling functions on dual Cantor sets*. The Proceedings of Symposia in Pure Mathematics, Vol. **48**, 1988.
- [SU4] D. Sullivan, *Bounds, quadratic differentials, and renormalization conjectures*. American Mathematical Society Centennial Publications, Volume **2**: Mathematics into the Twenty-First Century, AMS, Providence, RI, 1992, pp. 417-466.
- [SU5] D. Sullivan, *Conformal dynamical systems*. In Geometric Dynamics edited by J. Palis. Lecture Notes in Math. **1007**, Springer, 1983, pp. 725-752.
- [SU6] D. Sullivan, *Quasiconformal homeomorphisms in dynamics, topology, and geometry*. Proceedings of ICM, 1986, Berkeley, pp. 1216-1228.
- [TA1] L. Tan, *Similarity between the Mandelbrot set and Julia sets*. Commun. in Math. Phys., **134**, 1990, pp. 587-617.
- [TA2] L. Tan, *Voisinages connexes des points de Misiurewicz*. Ann. Inst. Fourier, Grenoble 124, 1992.
- [TAG] F. M. Tangerman, *Meromorphic continuation of Ruelle zeta functions*. Thesis, 1986, Boston University.
- [THU] W. Thurston, *Geometry and topology of three-manifolds*. Preprint, Princeton University, 1979.
- [TUK] P. Tukia, *Differentiability and rigidity of Möbius groups*. Invent. Math., **82**, 1985, pp. 557-578.
- [VET] J. J. P. Veerman and F. M. Tangerman, *A remark on Herman's theorem for circle diffeomorphisms*. IMS preprint 1990/13, SUNY at Stony Brook.
- [VSK] E. B. Vul, Ya. G. Sinai, and K. M. Khanin, *Feigenbaum universality and the thermodynamic formalism*. Russian Math. Surveys, Volume **39**, 1984, pp. 1-40.
- [YO1] J.-C. Yoccoz, *Conjugaison différentiable des difféomorphismes du cercle dont le nombre de rotation vérifie une condition Diophantienne*. Ann. Sci. Ec. Norm. Sup., **17**, 1984, pp. 333-361.
- [YO2] J.-C. Yoccoz, (1984), *Il n'y a pas de contre-exemple de Denjoy analytique*. C.R. Acad. Sci. Paris, **298**, série I, no. 7, 1984, pp. 141-144.
- [YO3] J.-C. Yoccoz, *Centralisateurs et conjugaison différentiable des difféomorphismes du cercle*. Thesis, 1985, Univ. Paris-Sud, Orsay.
- [YO4] J.-C. Yoccoz, *unpublished work*.
- [ZHL] M. Zhang and W. Li, *A rigidity phenomenon in the conjugacy for family of diffeomorphisms*. Preprint.
- [ZYG] A. Zygmund, *Smooth functions*. Duke Math. J., vol. **12**, 1945, pp. 47-76.

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