Math 131 Final Review Sheet
5/7/2010

Here is an itemized list of the material that could potentially be on the (cumulative) final exam. For each topic you can review the examples that I did in lecture, the worked out examples in the book, the two practice exams and midterms, past quizzes, and the homework solutions available on the course webpage. *While it is essential to know the concepts, try to spend most of your time doing problems rather than reading the book page-by-page.*

- Functions, the domain of a function, compositions of functions
- The graph of a function, the x and y intercepts, graphing quadratic functions, locating intersection points of two graphs
- Linear functions and the equation of a line, the point-slope formula
- Examples of mathematical modeling, the demand $D(x)$, revenue $R(x) = xD(x)$, cost $C(x)$ and profit $P(x) = R(x) - C(x)$, break-even analysis, law of supply and demand and how to find the equilibrium production level x_e and the equilibrium price p_e
- Limits, estimating limits by examining graphs or forming calculator tables, algebraic ways to find limits, limits involving infinity, one-sided limits
- Continuity and its geometric meaning (no "gaps" or "breaks" or "jumps" in the graph), the intermediate value property of continuous functions
- Definition of the derivative

  $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h},$$

  derivative as the slope of the tangent line, derivative as the rate of change
- The power formula

  $$(x^n)' = nx^{n-1}$$

  and the differentiation rules
  - Linearity: $(cf)' = cf'$ (c constant) and $(f \pm g)' = f' \pm g'$
  - Product rule: $(fg)' = f'g + g'f$
  - Quotient rule: $\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}$
  - Chain rule: $(f(g(x)))' = f'(g(x)) \cdot g'(x)$ or $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$
- Higher derivatives of a function, velocity $v(t)$ and acceleration $a(t)$ are the first and second derivatives of the position function $s(t)$

- The linear approximation formula
$$f(a+1) - f(a) \approx f'(a),$$
or more generally
$$f(a+h) - f(a) \approx hf'(a) \qquad (h \text{ small}),$$
applications in estimating marginal cost and revenue
- Implicit differentiation
- Related rates
- Increase and decrease of f and its relation to the sign of f', critical points and their type
- Concavity of f and its relation to the sign of f'', inflection points
- Curve sketching
- Optimization (how to maximize/minimize a quantity)
- The exponential and logarithm functions and their basic properties
- Application of the exponential function in the problem of compounding interest: with an initial investment P and the annual interest rate r compounded k times a year, the balance after t years is
$$B(t) = P\left(1 + \frac{r}{k}\right)^{kt}.$$
In the case of continuous compounding, the formula takes the form
$$B(t) = Pe^{rt}.$$
- Exponentially growing quantities $y = Ae^{kt}$, exponentially decaying quantities $y = Ae^{-kt}$, half-life and doubling time
$$T = \frac{\ln 2}{k}.$$
- Derivative of the exponential and logarithm functions:
$$(e^x)' = e^x \qquad (\ln x)' = \frac{1}{x}$$
applications in logarithmic differentiation.

Practice Problems

1. Find the limits:

(i) $\displaystyle\lim_{x\to 3} \frac{x^2 - 2x - 3}{x^2 - 9}$ (ii) $\displaystyle\lim_{x\to 4} \frac{\sqrt{x} - 2}{x - 4}$ (iii) $\displaystyle\lim_{x\to\infty} \frac{9x^5 + 3x^4 - 5x^3}{5x^3 - 3x^5}$

2. Find the derivative $y' = dy/dx$ of the following functions:

(i) $y = \sqrt{1 - 3x^3}$

(ii) $y = \left(1 - \frac{1}{x}\right)^{-4}$

(iii) $y = x^2 e^{-5x}$

(iv) $y = \frac{x^4 - 7}{\ln x}$

(v) $y = \frac{(x+2)^5}{\sqrt[6]{3x - 1}}$ (Hint: Logarithmic differentiation)

(vi) $xy^3 = 3x^2 + 2y^2 - 5$ (Hint: Implicit differentiation)

3. Let $f(x) = x^2 + 7x - 5$. Use the definition of the derivative to find $f'(x)$.

4. The unit price in dollars at which x units of a product are sold is

$$p(x) = -x^2 - 4x + 80$$

(i) Use marginal analysis to estimate the revenue generated from producing the 4th unit.

(ii) Find the actual revenue generated from producing the 4th unit.

5. A ladder 10 feet long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 foot per second, how fast is the top of the ladder sliding down the wall at the moment when the bottom of the ladder is 6 feet from the wall?

6. Consider the function $f(x) = x^4 + 4x^3 + 4x^2$. Find the formulas for f' and f'' and use them to determine the intervals of increase/decrease and concavity of f. Make sure you clearly identify the critical point(s) and their type (i.e., local max, local min, neither) as well as the inflection point(s). Using your findings, sketch the graph of f.

7. A Florida citrus grower estimates that if 60 orange trees are planted, the average yield per tree will be 400 oranges. The average yield will decrease by 4 oranges per tree for each additional tree planted. How many trees should the grower plant to maximize the total yield?

8. How much money should be invested at an annual interest rate of 5% to obtain a balance of $20,000 in 10 years if the interest is compounded quarterly? Continuously?

9. The half-life of radium is 1690 years. How long will it take for a 50-gram sample of radium to be reduced to 5 grams?