MAT 160, PROBLEM SEMINAR, WEEK OF 1/25/99

PROBLEM SET 1: PURE THOUGHT

Problem 1. Can you produce number 26 by using digits $2, 3, 4, 5$ once and symbols $+, -, \times, /$ at most once? Examples of permitted combinations are $(34/2) + 5 = 22$ and $(3/1) - 2 + (4 \times 5) = 21$, but rather than 22 or 21 we would like to construct 26.

Problem 2. If SAGE+SUAVE+SAGE=46933, what word will correspond to the number 46933? (*Hint*: The answer is a famous German mathematician, who lived from 1777 to 1855.)

Problem 3. Can you guess the next number in each sequence?
- $7, 26, 63, ?$
- $7, 26, 64, 140, ?$
- $61, 52, 63, 94, ?$

Problem 4. Must there be a Friday the 13th in every year?

Problem 5. When two digits in the base 10 representation of a natural number are interchanged, the difference between the old number and the new number is always divisible by 9. For example, $142 - 124 = 18 = 9 \times 2$ or $11205 - 21105 = -9900 = 9 \times -1100$. Can you explain why?

Problem 6. A spaceship is to travel between 4 planets A,B,C,D along the edges of the tetrahedron shown by dotted lines. Is it possible to plan the trip so as to traverse every edge exactly once?
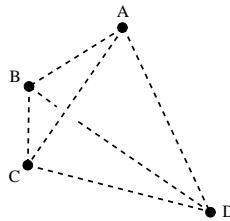
Problem 7. We remove the two diagonally opposite squares from an 8×8 chess-board, to get a board of 62 squares. Is it possible to cover this board with 31 non-overlapping domino pieces as in the picture? (*Hint*: Color the board as well as the pieces black and white.)