MAT 160, PROBLEM SEMINAR, WEEK OF 4/19/99

PROBLEM SET 10: SOME ELEMENTARY PROBABILITY

A couple of words on calculating probabilities:

- When the set of outcomes of an experiment is finite, the *probability* that the outcome satisfies a certain property P is by definition
$$\frac{\text{the number of outcomes which satisfy } P}{\text{the total number of possible outcomes}}$$
For example, if you roll two dice, the probability of getting 8 in the sum is computed as follows: The favorable outcomes are
$$\{6,2\},\{5,3\},\{4,4\},\{3,5\},\{2,6\}$$
and there are $6^2 = 36$ possible outcomes. So the probability is $5/36$ or almost 13%.
- If the outcomes of an experiment are points on a line segment I, assuming that the points are picked uniformly from I, the probability that the outcome belongs to a subset S of I is
$$\frac{\text{length}(S)}{\text{length}(I)}.$$
For example, if you pick numbers randomly and uniformly from the interval $[-1,1]$ on the real line, the probability that your number is larger than -0.3 is $1.3/2 = 0.65$.
- Similarly, if the outcomes of an experiment are points in a region R in the plane, assuming that the points are picked uniformly from R, the probability that the outcome belongs to a subset S of R is
$$\frac{\text{area}(S)}{\text{area}(R)}.$$
For example, if you pick points randomly and uniformly from the unit square, the probability that your point belongs to a sub-square of side length $1/2$ is $(1/4)/1 = 0.25$.

Problem 64. You roll three perfect dice. What is the probability of getting a sum of 10?

Problem 65. You roll four perfect dice. What is the probability of getting a sum of 16?

Problem 66. You pick real numbers randomly and uniformly from the interval $[0,1]$. Each such number has a unique non-terminating decimal expansion of the form $0.a_1a_2a_3\cdots$ (for example, $1/5 = 0.1999\cdots$). What is the probability that the number you pick has first digit $a_1 = 5$? What is the probability that the number you pick has second digit $a_2 = 7$?

Problem 67. Consider a dartboard of the shape of an equilateral triangle of side length 1 ft. You score *high* if the point you hit is not within distance 0.5 ft of any of the vertices. Assuming that you throw darts uniformly at the dartboard, what is your chance of scoring high?

Problem 68. A rare kind of rat produces a new baby rat which is either red with a chance of 60% or green with a chance of 40%. This new rat in turn produces a new rat which again is either red or green with the chances as above, and this process goes on until the 10th generation (the first baby rat being of generation 1). What is the probability that in this chain of 10 rats we have 7 red and 3 green rats? (*Hint*: Each possible chain of 10 rats corresponds to a word of length 10 consisting of letters R (for red) or G (for green). A favorable chain corresponds to a word with 7 R's and 3 G's, such as $RRRGGRGRRR$. Find the probability of getting any such chain and multiply it by the number of such favorable words.)

Problem 69. There are 4 guests at a party. At the end of the party, the host hands in their hats randomly without knowing which hat belongs to who. What is the probability that none of the guests gets back his own hat? (*Hint*: First solve the problem when there are 2 or 3 guests.)

Problem 70. Bunnie and Clyde have two children. One is a boy. With what probability is the other one a girl?