MAT 160, PROBLEM SEMINAR, WEEK OF 4/26/99

PROBLEM SET 11: GAMES AND MORE

You hardly need any background to attack the following problems.

Problem 71. Show how to cut this square into 5 connected pieces each of which contains all the vowels once only:

E	A	I	O	I
U	E	U	E	O
O	I	A	O	A
I	U	E	A	I
A	O	U	E	U

Problem 72. Two players take turns putting pennies on a round table, one penny per turn, without piling one penny on top of another. The player who cannot place a penny loses. Design a winning strategy for the first player.

Problem 73. Numbers 1 through 20 are written in a row. Two players take turns putting plus and minus signs between the numbers. When all such signs have been placed, the resulting number is evaluated (i.e., the additions and subtractions are performed). The first player will win if the resulting number is even, the second player will win if it is odd. Who will win and why?

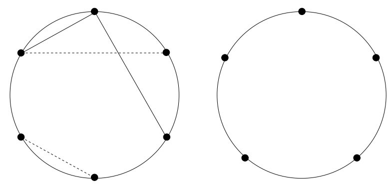
Problem 74. Mr. Vegas made 3 dice: one with the numbers 2,4,9 twice on its faces, one with the numbers 3,5,7 twice on its faces, and one with the numbers 1,6,8 twice on its faces. The total on the faces of each die is the same, but Mr. Vegas was confident that if he let his opponent choose a die first and roll it, he could select a die that would give him a better chance of rolling a higher number. Explain how this is possible.

Problem 75. *The Game of SIM.* This is a game for two players devised by Gustavus Simmons. Play starts with 6 points at the vertices of a hexagon inscribed in a circle. Two players using different colored pens take turns in joining two of these points with a line segment. The goal is to avoid drawing three lines that forms a triangle. The player who does this first loses.

(i) Try to play this game with a partner a few times to see how it works. Try to devise a winning strategy. Who wins most, the first or the second player?

(ii) It is also possible to play this game with only 5 points on the circle. What happens now? Can you find a winning strategy?

(iii) You will find that each time you play with 6 points, someone always loses. Can you find a simple argument to show that this game never ends in a draw? What happens with 5 points?



Problem 76. In a large urn there are 75 white and 150 black balls, and you also have a big pile of black balls. Now the following 2-step operation is performed repeatedly: First, two balls are withdrawn at random from the urn. If both are black, one is put back in the urn and one is thrown away. If both are white, both are thrown away and one black from the pile is put in the urn. Finally, if one is black and one is white, white is put back in the urn and black is thrown away. After successive applications, only one ball is left in the urn. What color is that ball?

Problem 77. On a chessboard, a rook stands on square a1. Two players take turns moving the rook as many squares as they want, either horizontally to the right or vertically upward. The player who can place the rook on square h8 is the winner. Design a winning strategy for the second player.