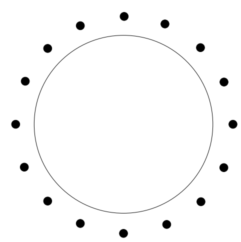
MAT 160, PROBLEM SEMINAR, WEEK OF 2/8/99

PROBLEM SET 3: THE PIGEONHOLE PRINCIPLE

Problem 15. There are N people at a party. Prove that you can find 2 of them who have the same number of friends among those present.

Problem 16. Of 16 tennis fans seated at a round table, more than half are women. Show that no matter how you arrange their position, there are always two women who are seated diametrically opposite to each other.



Problem 17. Consider 5 points inside a square of side length 1. Prove that you can choose 2 points among them whose distance is less than $1/\sqrt{2}$.

Problem 18. Given any set of 10 natural numbers between 1 and 99 (inclusive), prove that there are two disjoint subsets of the set with equal sums of their elements. (*Hint*: A set of 10 natural numbers has $2^{10} = 1024 > 1000$ subsets.)

Problem 19. A point (x, y) in the plane is called a *lattice point* if both coordinates x and y are integers (positive, negative, or zero). Suppose that we have 5 such lattice points and we connect them two by two by straight line segments. Prove that one of the resulting line segments must contain another lattice point. (*Hint*: Look at the midpoints of these segments.)

Problem 20. Given any set of 11 natural numbers between 1 and 20 (inclusive), show that at least one of the members of the set must divide another member of the set. (*Hint*: Note that each natural number can be written as a power of 2 times an odd number; for example, $8 = 2^3 \times 1$, $12 = 2^2 \times 3$, and $15 = 2^0 \times 15$. Apply this fact on each of the given 11 numbers.)

Problem 21. Any arrangement of 3 squares as shown in the picture is called a *trimino*. Given an 8×8 checkerboard, what is the maximum number of squares that you can color green such that it makes *no* trimino all green? (*Hint*: Divide the checkerboard into 16 disjoint 2×2 squares.)