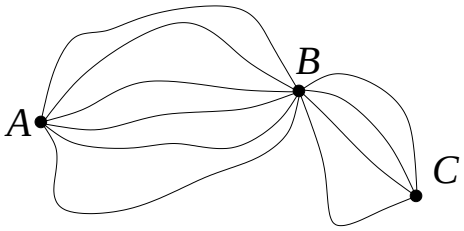
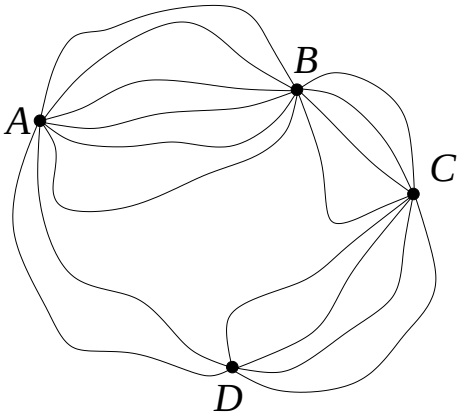
MAT 160, PROBLEM SEMINAR, WEEK OF 2/15/99

PROBLEM SET 4: ELEMENTARY COUNTING

Problem 22. (a) There are 3 towns A, B, C. 6 roads go from A to B and 4 roads go from B to C (see the left figure). In how many different ways can one go from A to C?

(b) A new town D and several new roads were built (see the right figure). How many different ways are there to go from A to C now?





Problem 23. Every student of the Pistachio Institute of Technology has a 4 digit identification number. Each digit can be one of $0, 1, 2, \cdots, 9$ and the id's may have repeated digits too, but they cannot start with a 0. At most how many students are there in this university?

Problem 24. Recall that for a non-negative integer n, the notation $n!$ (called "n factorial") is by definition the product $n(n-1)\cdots 3 \cdot 2 \cdot 1$, with the convention $0! = 1$. Simplify the following expressions:

$$n!(n+1) \qquad \frac{n!}{(n-1)!} \qquad \frac{(n+1)!}{n(n+1)}$$

Problem 25. Any finite sequence of letters is called a *word* (whether or not it makes sense). For each of the following examples, find the number of different words you can obtain by permuting the letters:

"VECTOR" "TRUST" "CARAVAN" "CLOSENESS"

Problem 26. How many 10 digit numbers have at least 2 digits equal?

Problem 27. Recall that for two sets A, B, the notation $A \cup B$ ("A union B") is the set of all elements which belong to either A or B or both, while $A \cap B$ ("A intersect B") is the set of all elements which belong to both A and B. Also $|A|$ means the number of elements in a set A.

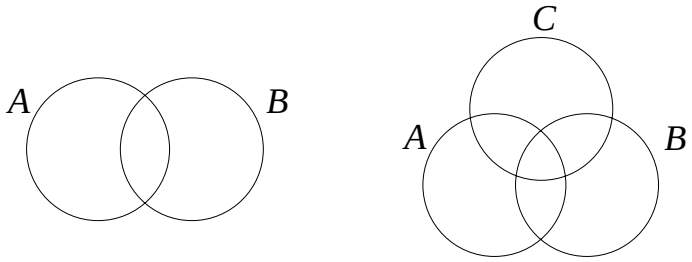
(a) If A and B are finite sets (each having a finite number of elements), show that

$$|A \cup B| = |A| + |B| - |A \cap B|.$$

(b) Similarly, for any three finite sets A, B, C, show that

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|.$$

(*Hint*: You may find it useful to look at the following diagrams)



Problem 28. There are 28 students in a class. Each one of them has either a red hat or a green shirt or a black coat (some of them have two or all three of these items). There are 10 students with a red hat, 15 students with a green shirt and 12 students with a black coat. Assume 5 students have both red hat and green shirt, 3 students have both red hat and black coat, and only 2 students have both green shirt and black coat. How many students have all three items? (*Hint*: Use part (b) of the previous problem.)