MAT 160, PROBLEM SEMINAR, WEEK OF 4/12/99

PROBLEM SET 9: BABY GEOMETRY

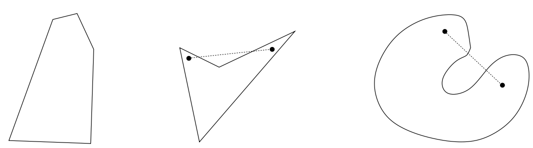
Let us recall a couple of facts from elementary Geometry:

• Among all paths connecting two points A and B in the plane, the straight line segment AB has the smallest possible length. We denote this length by $|AB|$.

• As a corollary, we have the so-called *Triangle Inequality*: For any three points A, B, and C in the plane,

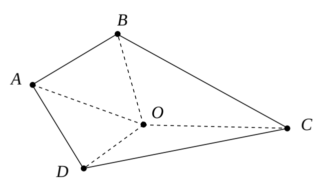
$$|AB| \leq |AC| + |CB|$$

• A region R in the plane in called *convex* if for any two points A and B in R, the entire line segment AB in contained in R. For example, a triangle is always convex. Here are further examples of convex and non-convex sets in the plane:

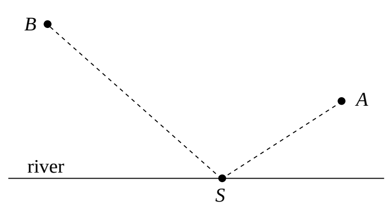


A convex pentagon A non-convex quadrilateral A non-convex region

Problem 57. Inside a convex quadrilateral $ABCD$ choose a point O such that the sum of the lengths $|OA| + |OB| + |OC| + |OD|$ is as small as possible.

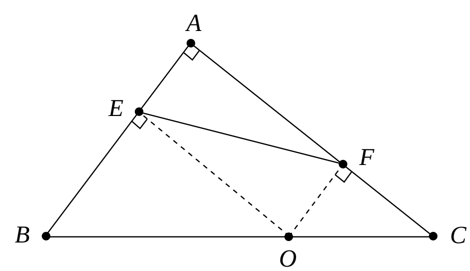


Problem 58. Towns A and B are on the same side of a river. A water pump station S is to be built somewhere by the river and be connected by straight pipelines to A and B. Where should S be built in order to make the total length of the piplines minimum? (*Hint*: Imagine what the situation would be if A and B were on the different sides of the river.)

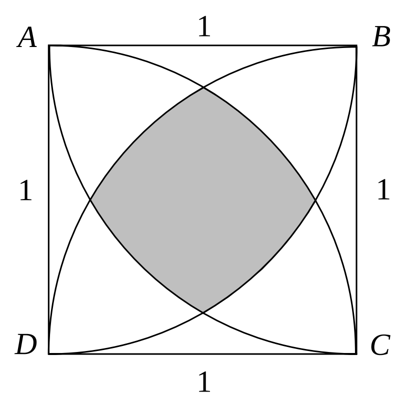


Problem 59. Prove that in every convex pentagon, the sum of the lengths of all five diagonals is greater than the perimeter but is less than twice the perimeter.

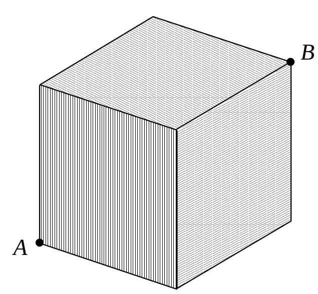
Problem 60. Consider a point O on the hypotenuse BC of a right triangle ABC. Draw perpendiculars OE and OF to AB and AC, respectively. Determine the position of O for which the length $|EF|$ is minimum. (*Hint*: $AEOF$ is a rectangle.)



Problem 61. Consider a square $ABCD$ of side length 1 and draw 1/4-circles of radii 1 centered at each of the points A, B, C, D as in the figure. Express the area of the resulting shaded region as a decimal number. (*Answer*: ≈ 0.315146.)



Problem 62. A bug is at the vertex A of a wooden cube of side length 1 and wants to reach vertex B. Which path makes its trip shortest?



Problem 63. The same bug in Problem 62 has to make a new journey: This time it is sitting at point A on a $3'' \times 3'' \times 10''$ piece of wood (see the figure) and wants to reach point B. Points A and B are located at heights $1''$ and $2''$ of the corresponding rectangular faces, respectively. Is it possible for the bug to make its trip shorter than $13''$?