Math 201 First Midterm Review Sheet

10/8/2009

Here is an itemized list of the material that the first midterm is based upon. Make sure you study them carefully. For each topic, review your lecture notes and the related section in the book, past quizzes, as well as the core problems/solutions that are available on the course webpage.

- Parametric curves in the plane, tangent lines to parametric curves, arc-length formula for parametric curves
- Polar coordinates, tangent lines to polar curves, areas and arc-lengths in polar coordinates
- 3-dimensional coordinate system, distance formula, equation of a sphere
- Vectors and their components, algebra of vectors, length of vectors
- The dot product and its properties, the angle between vectors, orthogonal vectors, projection of one vector along another
- The cross product and it properties, relation to the area of triangles
- Various forms of the equation of a line in space, parallel lines, equation of a plane in space, parallel and orthogonal planes, distance from a point to a plane
- Basic examples of quadric surfaces: cylinders, ellipsoids, paraboloids
- Vector functions and their derivatives, tangent vectors and unit tangent vectors, integration of vector functions
- Arc-length and curvature of parametric curves in space

Practice Problems

1. Consider the curve C in the plane defined by the parametric equations

$$x(t) = 3t^2 \qquad y(t) = 2t^3.$$

   (i) Sketch C and indicate how it is traced out as t goes from $-\infty$ to $+\infty$.

   (ii) Find the length of the part of C that corresponds to $0 \leq t \leq 2$.

2. Find the area enclosed by the inner loop of the polar curve $r = 1 - 2\sin\theta$.

3. Let $\mathbf{a} = \mathbf{i} + \mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$. Find

   (i) The projection of $\mathbf{a}$ in the direction of $\mathbf{b}$.

   (ii) The value of t such that $\mathbf{a}$ is orthogonal to $\mathbf{a} + t\mathbf{b}$.

4. Find the equation of the plane passing through the point $(1, -1, 1)$ which contains the line $x = 2y = 3z$.

5. Consider the curve C defined by the vector function

$$\mathbf{r}(t) = t^2\mathbf{i} + (t^3 - 5t + 1)\mathbf{j} + t^2\mathbf{k}.$$

(i) Find the equation of the line tangent to C at the point $(4, -1, 4)$.

(ii) Compute the curvature of C at the point $(4, -1, 4)$.