Math 201 Second Midterm Review Sheet

11/12/2009

Here is an itemized list of the material that the second midterm is based upon. Make sure you study them carefully. For each topic, review your lecture notes and the related section in the book, past quizzes, as well as the core problems/solutions that are available on the course webpage.

- Functions of several variables, their graph and level sets
- Partial derivatives, higher order partial derivatives, Clairaut's Theorem on equality of f_{xy} and f_{yx}.
- Tangent planes to a graph $z = f(x, y)$, linear approximation and differentials, applications in estimating quantities or errors
- Multivariable chain rule, application in implicit differentiation
- The gradient and its relation to directional derivatives, gradient points in the direction of fastest increase, gradient is orthogonal to level sets, using the gradient to find the tangent plane to surfaces in $\mathbb{R}^3$ defined as level sets
- Local vs. global (absolute) extrema, critical points, 2nd derivative test for determining the type of a critical point, finding the global extrema of a function over a bounded closed domain

Practice Problems

1. Use differentials to estimate the error in measuring the surface area S of a rectangular box of dimensions $x = 5$ cm by $y = 3$ cm by $z = 7$ cm, provided that x, y, z are measured with an error of no more than 0.05 cm. Is S more sensitive to the error in measuring x or y or z?

2. Suppose $w = f(x, y)$ is differentiable and we substitute $x = r\cos\theta,\ y = r\sin\theta$ (polar coordinates!) so w may be regarded as a function of r, θ.

(i) Compute the partial derivatives w_r, w_θ in terms of the partial derivatives f_x, f_y.

(ii) Show that

$$(w_r)^2 + \frac{1}{r^2}(w_\theta)^2 = |\nabla f|^2.$$

3. Find all the critical points of the function

$$f(x, y) = 2 + 2x + 2y - x^2 - y^2$$

and determine their type. Then find the absolute extrema of f over the triangular region in the first quadrant bounded by the lines $x = 0$, $y = 0$ and $y = 9 - x$.

4. In which direction does the function

$$f(x, y, z) = \ln(x^2 + y^2 - 1) + y + 6z$$

increase most rapidly at the point $(1, 1, 0)$? What is the maximum rate of increase at this point?

5. Show that the equation of the tangent plane to the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

at a point (x_0, y_0, z_0) can be written as

$$\frac{xx_0}{a^2} + \frac{yy_0}{b^2} + \frac{zz_0}{c^2} = 1.$$