Math 202 First Midterm Review Sheet

2/29/2024

Here is a list of the material that the first midterm is based on, including background material from Math 201 that you should know and be able to use. In addition to the sample problems below, you can review your lecture notes and the examples discussed in class, the worked-out examples in the book, and the homework problems and solutions available on WebAssign.

- basic algebra of vectors, dot product and cross product
- paths in space, velocity and acceleration vectors
- first and higher order partial derivatives, Clairaut's theorem on the equality of mixed partial derivatives, the chain rule
- gradient of a scalar function, critical points
- constrained extrema and Lagrange multipliers
- double integrals, Fubini's theorem, switching the order in an iterated integral, double integrals in polar coordinates
- triple integrals in Cartesian, cylindrical and spherical coordinates
- Jacobian of a transformation, the change of variables formula for double integrals

Practice Problems

1. Let $\mathbf{r}(t)$ be the position of a particle in space. Suppose the velocity vector $\mathbf{v}(t) = \mathbf{r}'(t)$ is orthogonal to the acceleration vector $\mathbf{a}(t) = \mathbf{v}'(t) = \mathbf{r}''(t)$ for all time t. Show that the particle's speed $v(t) = |\mathbf{v}(t)|$ is constant.

2. Let $f(x)$ be twice differentiable and c be a fixed number. Verify that the function

$$u(x,t) = \frac{1}{2}\Big(f(x+ct) + f(x-ct)\Big)$$

satisfies the *wave equation* $u_{tt} = c^2 u_{xx}$ with the initial condition $u(x,0) = f(x)$.

3. Let $\mathbf{r}$ denote the vector $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$. Show that

$$\nabla(|\mathbf{r}|^2) = 2\mathbf{r} \qquad \text{and} \qquad \nabla(|\mathbf{r}|) = \frac{\mathbf{r}}{|\mathbf{r}|}.$$

4. Use Lagrange multipliers to find the maximum value of xyz subject to the constraint $x^2 + 2y^2 + 3z^2 = 1$.

5. Compute the following multiple integrals:

(i) $\displaystyle\int_0^{\pi} \int_y^{\pi} \frac{\sin x}{x}\, dx\, dy.$

(ii) $\displaystyle\int_0^1 \int_0^1 \int_0^1 z\, e^{x+y}\, dx\, dy\, dz.$

6. Recall that the center of mass $(\overline{x}, \overline{y})$ of a uniform plate D in the plane is given by

$$\overline{x} = \frac{\displaystyle\iint_D x\, dA}{\text{area}(D)} \qquad \overline{y} = \frac{\displaystyle\iint_D y\, dA}{\text{area}(D)}.$$

Use polar coordinates to find the center of mass of the uniform quarter-disk D defined by the inequalities $x^2 + y^2 \leq R^2,\ x \geq 0,\ y \geq 0$.

7. Use an appropriate change of variables to evaluate $\displaystyle\iint_R x\, dx\, dy$, where R is the parallelogram with vertices at $(1,1), (4,2), (5,4), (2,3)$.