

Math 231 Practice Test, 9/28/2004

1. Consider the linear system

$$\begin{cases} 2x + 4y + 6z = 18 \\ 4x + 5y + 6z = 24 \\ 3x + y - 2z = 4 \end{cases}$$

- (i) Write this system as $A\mathbf{x} = \mathbf{b}$ by introducing the matrices A , \mathbf{x} and \mathbf{b} .
- (ii) Solve the system using Gauss-Jordan elimination (i.e., by reducing A to its reduced row echelon form).
- (iii) Find the inverse A^{-1} by applying elementary row operations on the identity matrix.
- (iv) Compute the product $A^{-1}\mathbf{b}$ and verify that it coincides with the solution you found in (ii).

2. Find all solutions of the linear system

$$\begin{cases} x + 3y + 2w = 2 \\ 2x + y + 5z + 4w = -16 \\ 2x + 3y + 3z = 4 \\ 3x + 11y - 2z + 11w = -1 \end{cases}$$

3. Find $\text{tr}(AB^{-1})$ if

$$A = \begin{bmatrix} 1 & 5 \\ -2 & 3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 7 & 5 \\ 3 & 2 \end{bmatrix}.$$

4. Let

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 5 \\ 0 & 3 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 4 & 5 & 11 \\ 2 & 1 & 5 \\ 1 & -1 & 0 \end{bmatrix}.$$

- (i) Explain how B can be obtained by applying two elementary row operations on A .
 - (ii) Find elementary matrices E_1 and E_2 such that $B = E_2E_1A$.
5. For what values of a, b, c is the following linear system consistent?

$$\begin{cases} x + 4y - 3z = a \\ 2x + 6y + 5z = b \\ x + 6y - 14z = c \end{cases}$$

6. Suppose

$$\det \begin{bmatrix} a & b & c \\ r & s & t \\ u & v & w \end{bmatrix} = 3.$$

Find

$$\det \begin{bmatrix} 3r & 3t & 3s \\ a - 2u & c - 2w & b - 2v \\ u & w & v \end{bmatrix}.$$

7. Suppose A and B are 5×5 matrices such that $\det A = 2$ and $\det B = -1$. Find $\det(2B^2A^{-1}B^t)$.

8. Suppose A is an $n \times n$ matrix for which 0 is an eigenvalue. Show that A is not invertible.