Math 231 Practice Test 1, 9/29/2005

1. Consider the linear system

$$\begin{cases} 2x + 4y + 6z = 18 \\ 4x + 5y + 6z = 24 \\ 3x + y - 2z = 4 \end{cases}$$

   (i) Write this system as $A\mathbf{x} = \mathbf{b}$ by introducing the matrices A, $\mathbf{x}$ and $\mathbf{b}$.
   (ii) Solve the system using Gauss-Jordan elimination (i.e., by reducing A to its reduced row-echelon form).
   (iii) Find the inverse A^{-1} by applying elementary row operations on the identity matrix.
   (iv) Compute the product $A^{-1}\mathbf{b}$ and verify that it coincides with the solution you found in (ii).

2. Find all solutions of the linear system

$$\begin{cases} x + 3y + 2w = 2 \\ 2x + y + 5z + 4w = -16 \\ 2x + 3y + 3z = 4 \\ 3x + 11y - 2z + 11w = -1 \end{cases}$$

3. Find $\operatorname{tr}(AB^{-1})$ if

$$A = \begin{bmatrix} 1 & 5 \\ -2 & 3 \end{bmatrix} \qquad \text{and} \qquad B = \begin{bmatrix} 7 & 5 \\ 3 & 2 \end{bmatrix}.$$

4. Let

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 5 \\ 0 & 3 & 1 \end{bmatrix} \qquad \text{and} \qquad B = \begin{bmatrix} 4 & 5 & 11 \\ 2 & 1 & 5 \\ 1 & -1 & 0 \end{bmatrix}.$$

   (i) Explain how B can be obtained by applying two elementary row operations on A.
   (ii) Find elementary matrices E_1 and E_2 such that $B = E_2E_1A$.

5. Under what condition(s) on a, b, c is the following linear system consistent?

$$\begin{cases} x + 4y - 3z = a \\ 2x + 6y + 5z = b \\ x + 6y - 14z = c \end{cases}$$

6. Suppose

$$\det \begin{bmatrix} a & b & c \\ r & s & t \\ u & v & w \end{bmatrix} = 3.$$

Find

$$\det \begin{bmatrix} 3r & 3t & 3s \\ a-2u & c-2w & b-2v \\ u & w & v \end{bmatrix}.$$

7. Suppose A and B are 5×5 matrices such that $\det A = 2$ and $\det B = -1$. Find $\det(2\, B^3 A^{-1} B^T)$.

8. True or false? Justify your answer.

- Every homogeneous system of 5 linear equations in 6 unknowns has infinitely many solutions.
- There are 2×2 matrices A and B such that $\mathrm{tr}(AB) \neq \mathrm{tr}(BA)$.
- If A is a square matrix such that $A^2 = A$, then either $A = 0$ or $A = I$.
- If A and B are square matrices such that $AB = 0$ and if A is invertible, then $B = 0$.