Math 231 Practice Test, 11/7/2004

1. Solve the system

$$\begin{cases} 2x + 3y - z = 1 \\ 3x + 5y + 2z = 8 \\ x - 2y - 3z = -1 \end{cases}$$

by Cramer's rule.

2. Consider the vectors $\mathbf{u} = (1, -1, 0, 2)$ and $\mathbf{v} = (0, 0, 1, 5)$ in $\mathbb{R}^4$. Find

(i) $\mathbf{u} \cdot (2\mathbf{u} - 3\mathbf{v})$
(ii) $\|\mathbf{u} + \mathbf{v}\|$
(iii) The unit vector in the direction of $\mathbf{u} - \mathbf{v}$.
(iv) The distance between $-\mathbf{u}$ and $\mathbf{v}$.
(v) The scalar $a \in \mathbb{R}$ such that $\mathbf{u}$ is orthogonal to $\mathbf{u} + a\mathbf{v}$.

3. Recall from Calculus that a function $f : \mathbb{R} \to \mathbb{R}$ is *odd* if $f(-x) = -f(x)$ for all $x \in \mathbb{R}$. Show that the set of all odd functions is a subspace of the vector space $\mathcal{F}(-\infty, +\infty)$.

4. Let $\{\mathbf{v}_1, \mathbf{v}_2\}$ be a basis for $\mathbb{R}^2$. Under what condition on real numbers a, b, c, d is $\{a\mathbf{v}_1 + b\mathbf{v}_2, c\mathbf{v}_1 + d\mathbf{v}_2\}$ also a basis?

5. Verify that

$$\mathcal{B} = \{\mathbf{v}_1 = (1, 1, 2, 0), \mathbf{v}_2 = (1, 0, 0, 0), \mathbf{v}_3 = (-2, 1, 3, 1), \mathbf{v}_4 = (2, -2, 0, 5)\}$$

is a basis for $\mathbb{R}^4$. Then find the coordinates of a vector $\mathbf{v} = (x, y, z, w)$ relative to $\mathcal{B}$.

6. Show that the set of all symmetric 3×3 matrices is a subspace of the vector space $\mathcal{M}_{3,3}$. What is the dimension of this subspace? Can you generalize this to symmetric $n \times n$ matrices?

7. Find a basis and dimension of the subspace W of $\mathbb{R}^3$ if

(i) $W = \{(x, y, z) \in \mathbb{R}^3 : x + 2y - z = 0\}$
(ii) $W = \{(x, y, z) \in \mathbb{R}^3 : x = y = -z\}$.

8. Consider the matrix

$$A = \begin{bmatrix} 1 & 3 & 1 & -2 & -3 \\ 1 & 4 & 3 & -1 & -4 \\ 2 & 3 & -4 & -7 & -3 \\ 3 & 8 & 1 & -7 & -8 \end{bmatrix}.$$

(i) Find bases for Row(A), Col(A) and Null(A).
(ii) Find rank(A) and nullity(A) and verify that rank(A) + nullity$(A) = 5$.
(iii) For what vectors $\mathbf{b} \in \mathbb{R}^4$ is the system $A\mathbf{x} = \mathbf{b}$ consistent?