

Math 231 Practice Test 2, 11/12/2005

1. Consider the vectors $\mathbf{u} = (1, -1, 0, 2)$ and $\mathbf{v} = (0, 0, 1, 5)$ in \mathbb{R}^4 . Find

- (i) $\mathbf{u} \cdot (2\mathbf{u} - 3\mathbf{v})$
- (ii) $\|\mathbf{u} + \mathbf{v}\|$
- (iii) The unit vector in the direction of $\mathbf{u} - \mathbf{v}$.
- (iv) The distance between \mathbf{u} and \mathbf{v} .
- (v) The scalar a such that \mathbf{u} is orthogonal to $\mathbf{u} + a\mathbf{v}$.

2. Recall from Calculus that a function f is *odd* if $f(-x) = -f(x)$ for all x . Show that the set of all odd functions is a subspace of the vector space $\mathcal{F}(-\infty, +\infty)$.

3. Suppose $\{\mathbf{v}_1, \mathbf{v}_2\}$ is a basis for \mathbb{R}^2 . Let

$$\mathbf{u}_1 = a\mathbf{v}_1 + b\mathbf{v}_2 \quad \mathbf{u}_2 = c\mathbf{v}_1 + d\mathbf{v}_2.$$

Under what condition on the scalars a, b, c, d is $\{\mathbf{u}_1, \mathbf{u}_2\}$ also a basis?

4. Verify that

$$\mathcal{B} = \{\mathbf{v}_1 = (1, 1, 2, 0), \mathbf{v}_2 = (1, 0, 0, 0), \mathbf{v}_3 = (-2, 1, 3, 1), \mathbf{v}_4 = (2, -2, 0, 5)\}$$

is a basis for \mathbb{R}^4 . Then find the coordinates of a generic vector $\mathbf{v} = (x, y, z, w)$ relative to \mathcal{B} .

5. Show that the set of all symmetric 3×3 matrices is a subspace of the vector space $\mathcal{M}_{3,3}$. Find a basis and dimension of this subspace.

6. Find a basis and dimension of the subspace W of \mathbb{R}^3 if

- (i) $W = \{(a, b, c) : a + 2b - c = 0\}$
- (ii) $W = \{(a, b, c) : a = b = -c\}$.

7. Consider the matrix

$$A = \begin{bmatrix} 1 & 3 & 1 & -2 & -3 \\ 1 & 4 & 3 & -1 & -4 \\ 2 & 3 & -4 & -7 & -3 \\ 3 & 8 & 1 & -7 & -8 \end{bmatrix}.$$

- (i) For what vectors \mathbf{b} in \mathbb{R}^4 is the system $A\mathbf{x} = \mathbf{b}$ consistent?
- (ii) Find bases for $\text{Row}(A)$, $\text{Col}(A)$ and $\text{Null}(A)$.
- (iii) Find $\text{rank}(A)$ and $\text{nullity}(A)$ and verify that $\text{rank}(A) + \text{nullity}(A) = 5$.