Math 319/320 Homework 1

Problem 1. Write (in words) the negation of each of the following statements:

(i) Jack and Jill are good drivers.
(ii) All roses are red.
(iii) Some real numbers do not have a square root.
(iv) If you are rich and famous, you are happy.

Problem 2. Provide a counterexample for each of the following statements:

(i) For every real number x, if $x^2 > 4$, then $x > 2$.
(ii) For every positive integer n, $n^2 + n + 41$ is a prime number.
(iii) No real number x satisfies $x + \frac{1}{x} = -2$.

Problem 3. Recall from calculus that a function f is *increasing* when the following condition holds:

"For all real numbers x and y, if $x \leq y$, then $f(x) \leq f(y)$."

(i) Explain precisely what it means for a function *not* to be increasing.
(ii) Using (i), show that the function $f(x) = x^3 - 3x$ is not increasing.

Problem 4. Show that if $\frac{x}{x-1} \leq 2$, then $x < 1$ or $x \geq 2$. (Hint: Assume $\frac{x}{x-1} \leq 2$ and $x \geq 1$, and conclude that $x \geq 2$.)

Problem 5. Recall that $n! = 1 \cdot 2 \cdots (n-1)n$. Use mathematical induction to show that

$$n! > 2^n$$

for all integers $n \geq 4$.

Problem 6. Use mathematical induction to show that

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$$

for all integers $n \geq 1$. Can you find a direct, induction-free proof of this? (See if you can come up with a "trick" to simplify the sum on the left.)