Math 328 Homework 10

due on Sunday 5/10/20

Problem 1. Let $f(x)$ have the Fourier transform $F(\omega)$. Express the Fourier transforms of the functions $f(2x - 1)$, $e^{i3x} f(x)$, and $f''(5x)$ in terms of $F(\omega)$.

Problem 2. Consider the function

$$f(x) = \begin{cases} e^{-x} & x \geq 0 \\ 0 & x < 0 \end{cases}.$$

(i) Find the Fourier transform $F(\omega)$.

(ii) Show that the convolution $(f * f)(x) = \int_{-\infty}^{\infty} f(y) f(x - y) \, dy$ is equal to $x f(x)$.

(iii) Find the Fourier transform of $f * f$ in two different ways: (a) By using the convolution theorem; (b) by using the fact that $x f(x)$ has the Fourier transform $i F'(\omega)$.

Problem 3. Verify that the convolution

$$(f * g)(x) = \int_{-\infty}^{\infty} f(y) g(x - y) \, dy$$

has the following properties:

(i) $f * g = g * f$.

(ii) $(f * g)' = f' * g = f * g'$ (here $'$ means differentiation).

(iii) $(f * g) * h = f * (g * h)$.