Math 328 Homework 1

due on Thursday 2/6/20

Problem 1.

(i) Find the general solution $u = u(x)$ of the first order linear ODE

$$x^2 u' + x u = 1.$$

Then write down a formula for the solution which satisfies the condition $u(1) = -3$.

(ii) Find the general solution $u = u(x)$ of the second order linear ODE

$$u'' + 2u' + 2u = 0.$$

Then write down a formula for the solution which satisfies the conditions $u(0) = u'(0) = 1$.

Problem 2. Find the general solution $u = u(x)$ of the homogeneous Cauchy-Euler equation

$$x^2 u'' - 3x u' + 4u = 0.$$

Problem 3. Verify that each of the following functions satisfies the given PDE:

(i) $u(x, y) = 3x^2 y - y^3$; $\qquad u_{xx} + u_{yy} = 0$

(ii) $u(x, t) = \sin(x - ct)$; $\qquad u_{tt} = c^2 u_{xx}$ (c is a constant)

(iii) $u(x, t) = \dfrac{1}{\sqrt{t}} \exp\left(-\dfrac{x^2}{4kt}\right)$; $\qquad u_t = k u_{xx}$ (k is a constant)