Math 328 Homework 3

due on Thursday 2/20/20

Problem 1. Find the Fourier series of the following functions:

(i) $f(x) = \cos x, \ -\infty < x < +\infty$.

(ii) $f(x) = \cos x, \ -\frac{\pi}{2} < x < \frac{\pi}{2}, \ f(x+\pi) = f(x)$ for all x.

(iii) $f(x) = \begin{cases} 0 & -1 < x < 0 \\ x & 0 < x < 1 \end{cases}, \quad f(x+2) = f(x)$ for all x.

Problem 2. Without computing the coefficients, explain why the Fourier series of the function

$$f(x) = \begin{cases} 0 & -L < x < 0 \\ 1 & 0 < x < L \end{cases}, \quad f(x+2L) = f(x) \text{ for all } x$$

has no cosine term in it.

Problem 3.

(i) Show that if $p(x)$ is a polynomial of degree n and $g(x)$ is a continuous function, then

$$\int p(x)g(x)\,dx = p(x)G_1(x) - p'(x)G_2(x) + p''(x)G_3(x) - \cdots + (-1)^n p^{(n)}(x)G_{n+1}(x).$$

Here, G_1 is an antiderivative of g, G_2 is an antiderivative of G_1, etc. (Hint: Differentiate both sides with respect to x).

(ii) Use (i) to show that for any constant $\lambda \neq 0$,

$$\int x^3 \cos(\lambda x)\,dx = \frac{x^3 \sin(\lambda x)}{\lambda} + \frac{3x^2 \cos(\lambda x)}{\lambda^2} - \frac{6x \sin(\lambda x)}{\lambda^3} - \frac{6 \cos(\lambda x)}{\lambda^4}.$$

(iii) Let $f(x) = x^3$ for $0 < x < \pi$. Sketch the graph of the even 2π-periodic extension of f over a few periods. Then use (ii) to compute the Fourier series of this extension.