Math 328 Homework 5

due on Thursday 3/5/20

Problem 1. Consider the function

$$f(x) = \begin{cases} C & 0 \le x < \frac{L}{2} \\ 0 & \frac{L}{2} \le x \le L, \end{cases}$$

where C and $L > 0$ are constants. Show that the solution of the heat equation

$$\begin{cases} u_t = k\, u_{xx} & 0 \le x \le L,\ t \ge 0 \\ u(x,0) = f(x) & 0 \le x \le L \\ u(0,t) = u(L,t) = 0 & t > 0 \end{cases}$$

is given by the series

$$u(x,t) = \frac{2C}{\pi} \sum_{n=1}^{\infty} \frac{1 - \cos(\frac{n\pi}{2})}{n} \exp\left(-\frac{k n^2 \pi^2}{L^2} t\right) \sin\left(\frac{n\pi x}{L}\right).$$

Problem 2. Two iron rods, each 20 cm long, are such that one is at temperature 100°C and the other at 0°C throughout. They are put in perfect contact at one end and their other ends are kept at 0°C. Use the result of problem 1 to show that after 600 seconds (= 10 minutes) the temperature at the point of contact is approximately 36°C. Assume the thermal diffusivity k of iron is 0.15 cm^2/sec.

Problem 3. Find the solution of the heat equation

$$\begin{cases} u_t = u_{xx} & 0 \le x \le 1,\ t \ge 0 \\ u(x,0) = x & 0 \le x \le 1 \\ u_x(0,t) = u_x(1,t) = 0 & t > 0. \end{cases}$$

Identify the steady-state and transient temperatures.