MATH 360 Take-Home Exam I

- **Total points: 20**
- **Solutions must be written completely, carefully and clearly.**
- **You are not allowed to share your ideas or solutions with others, but you may use your notes or any book on the subject.**
- **The deadline is 12:00 pm of Friday 10/19. Hand in your exams to me at DRL 4N59, or simply slip it under the door in case I'm not there.**

Problem 1. [4 points] Let $A \subset \mathbb{R}$ be a non-empty set which is bounded above. Define $B = \{x \in \mathbb{R} : -x \in A\}$. Show that

$$\inf(B) = -\sup(A).$$

Problem 2. [4 points] Show by induction that for every $n \in \mathbb{N}$

$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2.$$

Problem 3. [4 points] Suppose $\{x_n\}$ is a sequence of positive real numbers which is *not* bounded. Prove that the sequence $\{1/x_n\}$ has a subsequence which converges to zero. Show by an example that the sequence $\{1/x_n\}$ itself may not converge to zero.

Problem 4. [8 points] Each of the following statements is either true or false. Make a decision in each case, and convince me that your decision is right.

(i) If $|x + y| \leq 2$, then either $|x|$ or $|y|$ is at most 1.

(ii) No polynomial $f : \mathbb{R} \to \mathbb{R}$ of degree 4 can be one-to-one (a polynomial of degree 4 has the form $f(x) = ax^4 + bx^3 + cx^2 + dx + e$, where a, b, c, d, e are real numbers and $a \neq 0$).

(iii) If $\{x_n\}$ is a sequence in $\mathbb{R}$ such that $|x_n - 1| < 1$ for all n, then $\{x_n\}$ has a cluster point in the interval $[0, 2]$.

(iv) The sequence $\{x_n\}$ defined by $x_n = \sqrt{n+1} - \sqrt{n}$ does not converge.