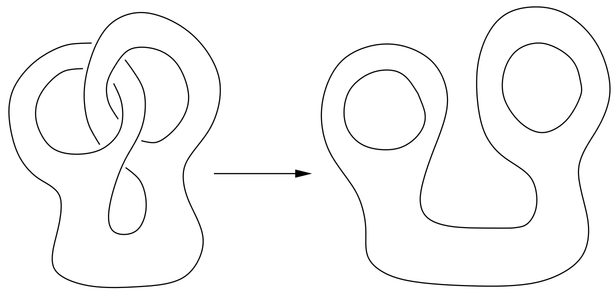
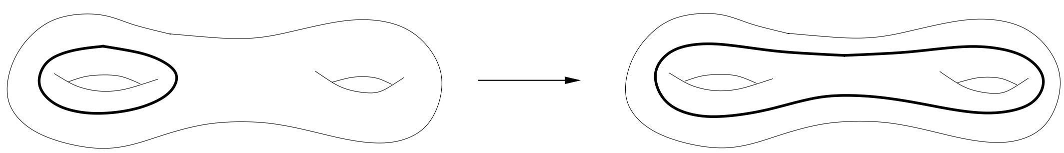
Math 364 Homework 1 (due 9/12/2002)

Problems 1 and 2 below are merely brain-teasers; solving them does not require any background in topology, or even mathematics.

Problem 1. A piece of gum is shown on the left. Explain by simple drawings how you can deform it to get the piece of gum shown on the right.



Problem 2. On a rubber donut with two holes, draw a loop going around one hole. You end up with what is shown on the left. Explain by simple drawings how you can deform it to get a similar donut with the loop going around both holes, as shown on the right.



Problem 3. Let us say that two letters of the English alphabet are *equivalent* if one can be obtained by continuously deforming the other (we consider lower cases, all letters have thickness, and imagine that they are written on a piece of rubber which can be deformed as much as you wish). For example, **a** and **b** are equivalent but **h** and **i** are not. Make a list of all equivalent letters.

Problem 4. (i) For which triples of points x, y, z on the real line does equality hold in the triangle inequality, that is

$$d(x, y) = d(x, z) + d(z, y)?$$

(ii) What would be the answer if x, y, z were points in the plane $\mathbb{R}^2$? What can you say about the general case $\mathbb{R}^n$?

Problem 5. For $j = 1, 2, 3, \ldots$, let A_j be the open interval $(2^{-j}, j)$ in $\mathbb{R}$. What is the union $\bigcup_{j=1}^{\infty} A_j$?

Problem 6. Give an example of an infinite collection $A_1, A_2, A_3, \ldots$ of open subsets of the real line such that the intersection $\bigcap_{j=1}^{\infty} A_j$ is not open.

Problem 7. Think of the real line $\mathbb{R}$ as the horizontal axis in the plane $\mathbb{R}^2$. Suppose $U \subset \mathbb{R}$ is open. Show that

$$U \times \mathbb{R} = \{(x, y) \in \mathbb{R}^2 : x \in U\}$$

is an open subset of the plane. (Hint: $U \times \mathbb{R}$ is the union of all vertical lines in $\mathbb{R}^2$ which intersect $\mathbb{R}$ at a point of U. Intuitively, if you pour blue paint only on the points of U and apply infinitely long vertical brush strokes, the resulting blue region in the plane will precisely be $U \times \mathbb{R}$.)

Reading assignment: Read Flegg's short informal chapter 3. Also read chapters 14 and 15 which help you remember the basic *language* of sets and functions. Feel free to skip pages 139 to 145 if you are not interested.