Math 364 Homework 6 (due 10/25/2002)

Problem 1. Show by simple drawings that all of the following diagrams represent the unknot:



Problem 2. Draw all possible knot diagrams with one or two crossings and verify that all of them represent the unknot. Conclude that there is no knot with crossing number 1 or 2.

Problem 3. Both of the following knot diagrams represent the right trefoil knot. Start with the diagram on the left and apply a sequence of Reidemeister moves to obtain the one on the right:



Problem 4. The following knot diagrams are mirror images of one another. Verify that both of them represent the same knot by showing that one diagram can be deformed to another.



Problem 5. Show that the following knot diagrams can be colored:



Problem 6. Determine whether or not the following knot diagram can be colored:



Problem 7. If a knot diagram is colorable, there are many different ways of coloring it. For example, strands that were colored red can be changed to green, green strands to blue, and blue strands to red. This will be a new coloring. There are 6 permutations of the set of 3 colors, so any coloring yields a total of 6 colorings. For some knots there are more possibilities.

- (i) Show that the standard diagram for the right trefoil knot (left picture in problem 3) has exactly 6 colorings.
- (ii) How many colorings does the following diagram admit?

