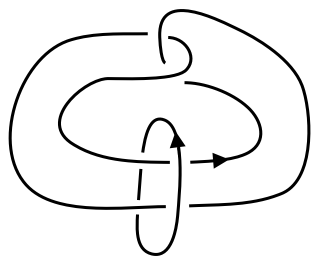
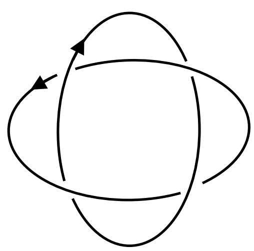
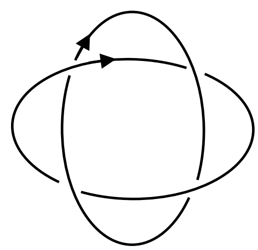
Math 364 Homework 8 (due 11/8/2002)

Problem 1. Compute the Conway polynomial of the Whitehead link represented by the following diagram:



Problem 2. The diagrams below show the King Solomon's knot and its mirror image. With the chosen orientations, both are 2-component links with 4 right crossings, so both have linking number 2. This means that the linking number cannot be used to tell them apart. Show, however, that these links have different Conway polynomials and hence are not equivalent.

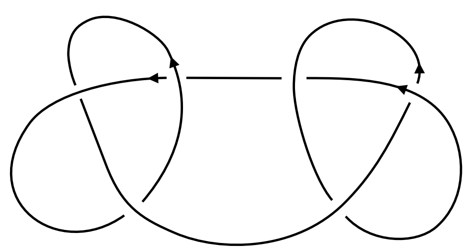




Problem 3. It is a theorem that

$$\nabla_{K_1 \# K_2} = \nabla_{K_1} \cdot \nabla_{K_2}.$$

In other words, the Conway polynomial of the connected sum of two knots is the product of their Conway polynomials. Verify this statement for the connected sum of a right and a left trefoil knot shown below:



Problem 4. Show that every open path-connected subset of the plane is a topological surface.

Problem 5. None of the following Euclidean sets is compact. In each case, explain why:

- $\{x \in \mathbb{R} : x = 1/n \text{ for some natural number } n\}$.
- $\{(x, y) \in \mathbb{R}^2 : |xy| \leq 1\}$.
- The sphere in $\mathbb{R}^3$ with the north and south poles removed.

Problem 6. Take a rectangular strip (for convenience choose one whose horizontal edges are much longer than the vertical edges). Gluing the vertical edges with no twist will make a cylinder, while gluing them after a half-twist will make a Möbius band.

(i) What will happen if you glue the vertical edges after a full twist? Is what you get orientable? Is it homeomorphic to something familiar? What does the boundary of it look like?

(ii) Answer the same questions when you glue the vertical edges after three half-twists.

(Hint: Experiment with an actual paper strip! After making each model, draw a simple picture of it. It will especially help you see what the boundary of each model looks like.)

Problem 7. Take a rectangular strip as shown and after a half-twist glue the vertical edges to obtain a Möbius band. Then cut along the midline (shown in dots). What kind of object do you end up with?