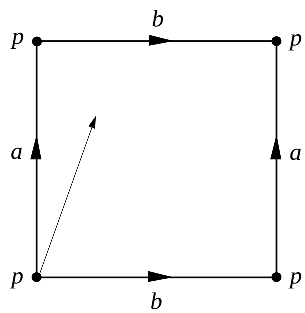
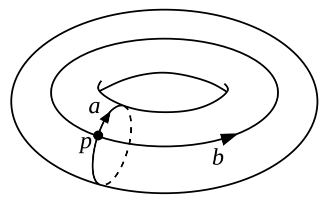
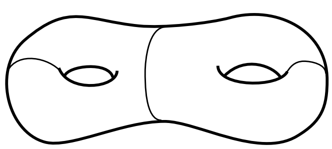
Math 364 Homework 9 (due 11/22/2002)

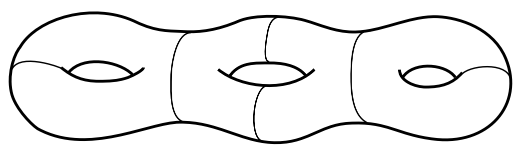
Problem 1. Let S be the square with its opposite edges identified, as shown on the left. Then S is homeomorphic to the torus $\mathbb{T}^2$ shown on the right. Suppose you are at the point p of S and you shoot a ball at a slope of 3 relative to the horizontal direction. Draw the trajectory of this ball in S, and then draw the same trajectory on the surface of $\mathbb{T}^2$.



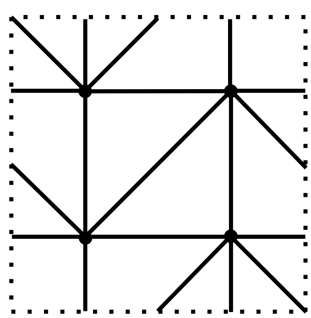


Problem 2. The following figures show standard "pant decompositions" of the 2-hole and 3-hole tori, as discussed in class. Can you think of other ways of decomposing these surfaces into the same number of pants?





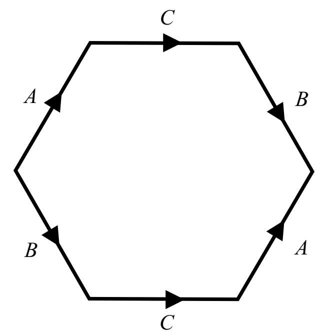
Problem 3. The following diagram shows a triangulation of the square model of the torus. Label the vertices, edges, and faces. How many of each are there?



Problem 4. Triangulate the octagonal model of the 2-hole torus in any way you want. Count the number of vertices, edges, and faces of your triangulation.

Problem 5. Triangulate the square model of the Klein bottle in any way you want. Count the number of vertices, edges, and faces of your triangulation.

Problem 6. Show by simple drawings that gluing the opposite edges of a hexagon as suggested below produces the surface of a torus:



Problem 7. Divide the surface of a torus into seven countries such that every two countries become neighbors (i.e., share some border). Perhaps it is easier to work on the hexagonal model of the torus suggested in the previous problem.