Question 1.

Use the simple (neoclassical) model we saw in class (with the Cobb-Douglas production function) to predict the impact on the real wage and on the real rental price of capital for each of the following events (shocks). Argue your answers graphically.

a) *The labor market participation of women increases.*

This is an increase in the labor force (L-bar). As a result the supply curve shifts to the right in the labor market and the demand curve shifts to the right in the capital market. Compared to the initial equilibrium, the new equilibrium features a lower real wage and a higher real rental rate for capital. See the figure.

b) *An earthquake destroys part of the capital stock.*

This is a reduction in the stock of capital (K-bar). The supply curve in the capital market shifts to the left. The demand curve in the labor market shifts to the left as well. In the new equilibrium the real rental rate for capital is higher and the real wage is lower.

c) *Technological progress improves the production function.*

Consider a Cobb-Douglas production function \( F(K,L) = AK^{0.5} L^{0.5} \) and suppose there is an increase in the total factor productivity term (A) due to technological progress. Compute the MPL and MPK functions. Clearly, an increase in A will increase the MPL and MPK at any given values of L and K. Thus there is a shift upward in both the demand for labor and the demand for capital. In the new equilibrium the real wage and the real rental rate for capital are higher than initially.
QUESTION 1.

a) $L$ increases.

b) $K$ falls.

c) Both MPL and MPK shift upward.
**Question 2.** Consider the following (linear) production function:

\[ Y = F(K,L) = 10(K + 2L). \]

*a) Is it constant returns to scale? Is it neoclassical?*

Yes, it is CRS. To prove this suppose we double the amount of inputs used in production. That is, for any given amount of labor and capital,

\[ F(2K,2L) = 10(2K + 4L) = 2 \times 10(K + 2L) = 2F(K,L), \]

That is, the level of output exactly doubles. This is the definition of CRS.

But this production function is not neoclassical. Its MPK function is \( MPK = 10 \), which clearly is not a decreasing function of \( K \). Hence, this production function does not satisfy all the requirements to be a neoclassical production function.

*b) Calculate the MPL.*

The partial derivative of \( F(K,L) \) with respect to \( L \) equals 20. Alternatively, for the linear production function we can apply the following formula:

\[ MPL = F(K,L+1) - F(K,L) = 20, \]

which leads to the same answer.

*c) Does the MPL decrease as \( L \) increases? Does the MPL increase as \( K \) increases?*

No, they are fixed numbers.
**Question 3.** Consider the following simple neoclassical economy with a full employment level of production (Y-bar in the slides) of 5,000 units of output. Moreover, fiscal policy and the consumption and investment functions are:

\[ G = 1,000 \text{ and } T = 1,000 \]
\[ C = 250 + 0.75(Y - T) \]
\[ I = 1,000 - 50r \]

a) Compute private savings, public savings and national savings. Also compute the consumption and investment levels.

We can compute public savings right away \((T - G) = 0.\)

In general (when the consumption function depends on the interest rate) to compute private savings, consumption and investment we first need to know the equilibrium real interest rate. However, in this particular case we can easily compute private and national savings and the level of consumption in equilibrium.

\[ C = 250 + 0.75(Y - T) = 3250. \]
\[ S^p = Y - T - C = 4000 - 3250 = 750. \]
\[ S = S^p + S^G = 750. \]

b) Find the equilibrium real interest rate.

We can either use the condition for supply equal demand in the output market or in the loans market. Both lead to the same answer. Let’s use the output market:

\[ Y = C + I + G \]
\[ 5000 = 3250 + (1000 - 50r) + 1000 \]
\[ 5000 = 5250 - 50r \]
\[ 50r = 250 \]
\[ r^* = 5. \]

We can now compute the equilibrium level of investment:
\[ I^* = 1000 - 250 = 750. \]

c) Now suppose that \(G\) rises to 1,250. Compute private savings and national savings.

\[ C = 250 + 0.75(Y - T) = 3250. \]
\[ S^p = Y - T - C = 4000 - 3250 = 750. \]
\[ S = S^p + S^G = 750 - 250 = 500. \]
d) Find the new equilibrium interest rate. Compute consumption and investment. Compare these values to the equilibrium with the lower government purchases.

\[ Y = C + I + G \]
\[ 5000 = 3250 + (1000 - 50r) + 1250 \]
\[ 5000 = 5500 - 50r \]
\[ 50r = 500 \]
\[ r^* = 10. \]

We can now compute the equilibrium level of investment:
\[ I^* = 1000 - 5000 = 500. \]

The increase in government purchases (by 250) has reduced private-sector spending (C+I) in exactly the same amount (complete crowding out). The mechanism has been the increase in the real interest rate driven by the lower public and national savings.

In general (when the consumption function also depends on the real interest rate) both consumption and investment will fall. But in this example only investment responds to changes in the interest rate. Therefore investment takes all the hit and falls by 250, while consumption remains unchanged.
Question 4

Read the NYTimes article “Prune and Grow” by David Brooks. Answer the following questions.

**a) What is the effect on income and employment of a fiscal stimulus in the neoclassical model that we have studied in class? And on real wages?**

In our simple neoclassical model fiscal policy has no effects on the level of income, employment or real wages. It only affects the composition of spending (complete crowding out).

**b) Summarize the study by professor Glaeser on the relationship between stimulus money and employment growth at the state level. What is the exercise he performed? What did he find?**

This study compares employment growth at the state level with the amount of stimulus money received by each state. He found only weak evidence that more fiscal stimulus in a state leads to higher employment growth.

**c) Find out (in the textbook or in Wikipedia) what the Ricardian equivalence is. What does it say about increases in G or tax cuts today and their current effect on consumer spending?**

Consumers understand that the government has an intertemporal budget constraint. In practice, today’s deficits need to be financed by taxes in the future. It is thus possible that an increase in G (or tax cut) today leads consumers to reduce their spending (C) in order to save enough to pay the future increase in taxes. In other words, this can also explain why a fiscal stimulus may fail to increase income and spending in addition to crowding out.

**d) What does professor Alesina’s study say about the effects of reducing public debt on subsequent economic growth?**

Alesina’s study suggests that increases in public saving (deficit reductions) are associated to higher future economic growth.
**Question 5.** Consider the following Cobb-Douglas production function:

\[ F(K,L) = 10K^{1/3}L^{2/3} \]

a) Compute the marginal product of labor by taking the partial derivative with respect to \( L \).

\[ MPL = \frac{20}{3} \left( \frac{K}{L} \right)^{1/3} \]

b) Compute the marginal product of capital by taking the partial derivative with respect to \( K \).

\[ MPK = \frac{10}{3} \left( \frac{K}{L} \right)^{2/3} \]

c) Suppose that initially \( K=1 \) and \( L=8 \) are being used in production. Compute the resulting units of output. Now suppose the firm uses \( K=2 \) and \( L=16 \) in production. Compute the new level of output.

\[ F(1,8) = 40. \]
\[ F(2,16) = 80. \]

d) What does this say about the returns to scale in this production function? Are they increasing, decreasing or constant?

Both inputs have been doubled and output has exactly doubled. Hence, this production function exhibits constant returns to scale.