6.1 Introduction

- In this chapter, we learn:
  - New methods of using existing resources are the key to sustained long-run growth.
  - Why "nonrivalry" makes ideas different from other economic goods in a crucial way.
  - How the economics of ideas involves increasing returns and leads to problems with Adam Smith’s invisible hand.

- The Romer model of economic growth.

- How to combine the Romer and Solow models to get a full theory of long-run economic performance.

6.2 The Economics of Ideas

- Adam Smith’s invisible hand theorem states that perfectly competitive markets lead to the best of all possible worlds.

- Idea diagram:

  ![Idea Diagram](image)

- Ideas in the world
  - Are virtually infinite

- Objects in the world
  - Are finite

- Sustained economic growth occurs because of new ideas.
Nonrivalry

- Objects are rivalrous
  - One person’s use reduces their inherent usefulness to someone else.

- Ideas are nonrivalrous
  - One person’s use does not reduce their inherent usefulness to someone else.
  - Nonrivalry implies we do not need to reinvent ideas for additional use.

Nonrivalry is different from excludability.

Excludability
- Someone may legally restrict use of a good.
- Ideas may be excludable.

Increasing Returns

- Firms pay initial fixed costs to create new ideas but don’t need to reinvent the idea again later.

- Increasing returns to scale
  - A doubling of inputs will result in a more than doubling of outputs.

- Constant returns to scale
  - Average production per dollar spent is constant.
  - Doubling inputs exactly doubles output.
  - The standard replication argument implies constant returns to scale.

- Increasing returns to scale
  - Average production per dollar spent is rising as the scale of production increases.

Test for increasing returns
- Multiplying all inputs by two
- Increasing returns is present if output is then multiplied by more than two.

\[ y_t = f(k_t, l_t, a_t) = a_t k_t^{1/3} l_t^{2/3} \]

\[ f(2k, 2l, 2a) = 2^4 f(k, l, a) = 2 \cdot 2^{1/3} \cdot 2^{2/3} \cdot a \cdot k^{1/3} l^{2/3} \]

\[ = 4 \cdot a \cdot k^{1/3} l^{2/3} = 4 \cdot f(k, l, a) \]

\[ 4 > 2 \]

increasing returns
Problems with Pure Competition

- Pareto optimal allocation
  - There is no way to change an allocation to make someone better off without making someone else worse off.
  - Perfect competition results in Pareto optimality because \( P = MC \).

- Under increasing returns to scale, a firm faces
  - Initial fixed costs
  - Marginal costs
- If \( P = MC \) under increasing returns, no firm will do research to invent new ideas.
  - The fixed research costs will never be recovered.

- Patents
  - Grant monopoly power over a good for a period
  - Generate positive profits
  - Provide incentive for innovation
- However, \( P > MC \) results in welfare loss.
- Other incentives for creating ideas may avoid welfare loss.
  - Government funding
  - Prizes

Case Study: Open Source Software and Altruism

- Profits are not the only way of encouraging innovation.
- Other motives:
  - Altruistic generosity
  - Desire to signal skills
  - “Purpose motives”

Case Study: Intellectual Property Rights in Developing Countries

- Why would poor countries ignore intellectual property rights?
  - Items or ideas obtained cheaply
  - May encourage multinational firms to relocate to developing countries

6.3 The Romer Model

- The Romer model
  - Focuses on the distinction between ideas and objects
  - Yields four equations
  - Stipulates that output requires knowledge and labor
- The production function of the Romer model
  - Constant returns to scale in objects alone
  - Increasing returns to scale in objects and ideas
• New ideas depend on
  – The existence of ideas in the previous period
  – The number of workers producing ideas
  – Worker productivity
  – Unregulated markets traditionally do not provide enough resources to produce ideas—and hence they are underprovided.
• The population
  – Workers producing ideas and workers producing output

Endogenous variables

\[ L_{at} = \bar{\ell} \bar{L} \]

\[ L_{yi} = (1 - \bar{\ell})\bar{L} \]

Unknowns/endogenous variables: \( Y_t, A_t, L_{yt}, L_{at} \)
Output production function \( Y_t = A_t L_{yt} \)

Innovation function: \( \Delta A_{t+1} = \bar{z}A_t L_{at} \)
Resource constraint \( L_{yt} + L_{at} = \bar{L} \)
Allocation of labor \( L_{at} = \bar{L} \)

Parameters: \( \bar{z}, \bar{L}, \bar{\ell}, A_0 \)

---

Recall that to solve a model we express all the endogenous variables in terms of the parameters and time.

Solving the Romer Model

• Romer model:
  – Output per person depends on the total stock of knowledge.
  \[ y_t = \frac{Y_t}{L} = A_t (1 - \bar{\ell}) \]

• Solow model:
  – Output per person depends on capital per person.

\[ \Delta A_{t+1} = \bar{z}A_t L_{at} = \bar{z} \bar{\ell} \bar{L} \]
• The stock of knowledge depends on its initial value and its growth rate.

\[
A_t = A_0 (1 + \bar{g})^t
\]

\[\bar{g} \equiv \bar{z} \bar{\ell} \bar{L}\]

Why is There Growth in the Romer Model?

• The Romer model produces the desired long-run economic growth that Solow did not.

• In the Solow model, capital has diminishing returns:
  \(\text{Eventually, capital and income stop growing.}\)

• The Romer model does not have diminishing returns to ideas because they are nonrivalous.

• Look at the exponents on the endogenous terms on the right side:
  \(\text{Labor and ideas have increasing returns together.}\)
  \(\text{Returns to ideas are unrestricted.}\)

• The Romer model does not have diminishing returns to ideas because they are nonrivalous.

\[
\Delta A_{t+1} = \bar{z} A_t \bar{L} \Delta t
\]

Balanced Growth

• The Solow model
  \(-\text{Transition dynamics}\)

• The Romer model
  \(-\text{Does not exhibit transition dynamics}\)
  \(-\text{Instead, has balanced growth path.}\)
  \(-\text{The growth rates of all endogenous variables are constant.}\)

\[\bar{g} = \bar{z} \bar{\ell} \bar{L}\]
Case Study: A Model of World Knowledge

- The United States has more researchers than Luxembourg has people.
- Growth rates 1960–2007
  - United States
    - 2.3 percent per year increase in per capita GDP
  - Luxembourg
    - 3.2 percent per year increase in per capita GDP
- How?
  - All countries can benefit from all ideas, no matter where the ideas were discovered.

Experiments in the Romer Model

**Parameters in the Romer model:**

\[ y_t = \bar{A}_0 (1 - \bar{\ell})(1 + \bar{g})t \]

- Initial stock of ideas at time \( t = 0 \)
- Fraction of the population doing research
- Productivity
- Population

**Experiment #1: Changing the Population**

\[ y_t = \bar{A}_0 (1 - \bar{\ell})(1 + \bar{g})t \]

- \( \bar{g} = \bar{z}\bar{\ell}\bar{L} \)
- A change in population changes the growth rate of knowledge.
- An increase in population will immediately and permanently raise the growth rate of per capita output.

**Experiment #2: Changing the Research Share**

\[ y_t = \bar{A}_0 (1 - \bar{\ell})(1 + \bar{g})t \]

- \( \bar{g} = \bar{z}\bar{\ell}\bar{L} \)
- An increase in the fraction of labor making ideas, holding all other parameters equal, will increase the growth rate of knowledge.

- If more people work to produce ideas, less people produce output.
  - The level of output per capita jumps down initially.
- But the growth rate has increased for all future years.
  - Output per person will be higher in the long run.
Growth Effects versus Level Effects

- The exponent on ideas in the production function
  - Determines the returns to ideas alone
- If the exponent on ideas is not equal to 1:
  - The Romer model will still generate sustained growth.
  - Growth effects are eliminated if the exponent on ideas is less than 1.
    - due to diminishing returns

Case Study: Globalization and Ideas

- Consequences of globalization
  - Ideas can be shared more easily.
  - More gains from trade realized.
  - More technologies will come from developing economies.

6.4 Combining Solow and Romer: Overview

- The combined Solow-Romer model
  - Nonrivalry of ideas results in long-run growth along a balanced growth path
  - Exhibits transition dynamics if economy is not on its balanced growth path
    - For short periods of time
      - Countries can grow at different rates.
    - In the long run
      - Countries grow at the same rate.

6.5 Growth Accounting

- Growth accounting determines
  - The sources of growth in an economy
  - How they may change over time
- Consider a production function that includes both capital ($K_t$) and ideas ($A_t$).

\[ Y_t = A_t K_t^{1/3} L_t^{2/3} \]

- The stock of ideas ($A_t$) is referred to as total factor productivity (TFP).
• Apply growth rate rules to the production function.
  – Growth rate version of the production function
  – The growth rate of each input weighted by its exponent

\[
g_{Yt} = g_{A_t} + \frac{1}{3}g_{Kt} + \frac{2}{3}g_{Lt}
\]

<table>
<thead>
<tr>
<th>Growth rate of output</th>
<th>Growth rate of knowledge</th>
<th>Growth contribution from capital</th>
<th>Growth contribution from workers</th>
</tr>
</thead>
</table>

• Adjust growth rates by labor hours.

\[
g_{Yt} - g_{L_t} = \frac{1}{3} (g_{Kt} - g_{L_t}) + \frac{2}{3} (g_{Lt} - g_{Lt}) + g_{Lt}
\]

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Output per hour, $Y/L$</td>
<td>2.5</td>
<td>1.8</td>
</tr>
<tr>
<td>Contribution of $K/L$</td>
<td>0.6</td>
<td>0.5</td>
</tr>
<tr>
<td>Contribution of labor composition</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>Contribution of TFP, $A$</td>
<td>1.4</td>
<td>2.2</td>
</tr>
</tbody>
</table>

**Table 6.2: Growth Accounting for the United States**

6.6 Concluding Our Study of Long-Run Growth

• Institutions (property rights, laws) play an important role in economic growth.
• The Solow and Romer models
  – Provide a basis for analyzing differences in growth across countries.
  – Do not answer why investment rates and TFP differ across countries.

From 1973–95
– Output in the United States grew half as fast as from 1948–73.
– This slower era of growth is known as the productivity slowdown.

From 1995–2002
– Output grew nearly as rapidly as before the productivity slowdown.
– This recent era is known as the new economy.

Case Study: Institutions, Ideas, and Charter Cities

• Institutions
  – Are nonrival
  – May help the poorest countries, even though many haven’t adopted this idea yet

• Charter Cities
  – Economy agrees to set the rules by which a new city is administered.
  – Hong Kong
6.7 A Postscript on Solow and Romer

• The Solow and Romer models have made many additional valuable contributions:
  – The modern theory of monopolistic competition
  – New understanding of exogenous technological progress

6.8 Additional Resources

• See the text for additional resources on ideas, institutions, and economic growth.

Summary

• Solow
  – Divides the world into capital and labor
• Romer
  – Divides the world into ideas and objects
• This distinction proves to be essential for understanding the engine of growth.

• Ideas
  – Are instructions for using objects in different ways
  – Are nonrivalrous; they are not scarce in the same way that objects are
  – Can be used by any number of people simultaneously without anyone’s use being degraded

• This nonrivalry implies
  – The economy is characterized by increasing returns to ideas and objects taken together.
• There are fixed costs associated with research (finding new ideas).
  – A reflection of the increasing returns

• Increasing returns imply that Adam Smith’s invisible hand may not lead to the best of all possible worlds.
• Prices must be above marginal cost in some places in order for firms to recoup the fixed cost of research.
• In the Solow model
  – Growth eventually ceases because capital runs into diminishing returns.

• In the Romer model
  – Because of nonrivalry, ideas need not run into diminishing returns.
  – This allows growth to be sustained.

• Combining the insights from Solow and Romer leads to a rich theory of economic growth.

• The growth of world knowledge explains the underlying upward trend in incomes.

• Countries may grow faster or slower than this world trend because of the principle of transition dynamics.

6.9 Appendix: Combining Solow and Romer (Algebraically)

• The combined model is set up by adding capital into the Romer model production function.

Additional Figures for Worked Problems

Setting Up the Combined Model

• The combined model features five equations and five unknowns.

• The five unknowns
  – Output $Y_t$
  – Capital $K_t$
  – Knowledge $A_t$
  – Workers $L_t$
  – Researchers $L_{at}$

The equations are:

\[ Y_t = A_t K_t^{1/3} L_t^{2/3}, \]

\[ \Delta K_{t+1} = \bar{\delta} Y_t - \bar{d} K_t, \]

\[ \Delta A_{t+1} = \bar{\xi} A_t L_{at}, \]

\[ L_{yt} + L_{at} = \bar{L}, \]

\[ L_{at} = \bar{k} \bar{L}. \]
The production function for output
\[ Y_t = A_t K_t^{1/3} L_t^{2/3}, \]

The accumulation of capital over time
\[ \Delta K_{t+1} = \delta Y_t - \delta K_t, \]

Ideas
\[ \Delta A_{t+1} = \bar{z} A_t L_{at}, \]

The numbers of workers and researchers sum to equal the total population.
\[ L_{yt} + L_{at} = \overline{L}, \]

Our assumption that a constant fraction of the population works as researchers
\[ L_{at} = \overline{\ell} \overline{L}. \]

The production function will have constant returns to scale in objects, but increasing returns in ideas and objects together.
\[ Y_t = A_t K_t^{1/3} L_t^{2/3}, \]

The change in the capital stock is investment minus depreciation.
\[ \Delta K_{t+1} = \delta Y_t - \overline{d} K_t, \]

Researchers are used to produce new ideas.
\[ \Delta A_{t+1} = \bar{z} A_t L_{at}. \]

Solving the Combined Model

- The combined model will result in:
  - A balanced growth path
    - (since \( A_t \) increases continually over time)
  - Transition dynamics

Long-Run Growth

- To be on a balanced growth path, output, capital, and stock of ideas all must grow at constant rates.

Start with the production function for output and apply the rules for computing growth rates:
\[ g_{Yt} = g_{At} + \frac{1}{3} g_{Kt} + \frac{2}{3} g_{Lyt} \]

\[ g_{Yt} = \Delta Y_{t+1} / Y_t \]
To solve for the growth rate of knowledge
  – Divide the production function for new ideas by $A_t$
    
    $g_{At} = \frac{\Delta A_{t+1}}{A_t} = \bar{z}L_{at} = \bar{z}L$

To solve for the growth rate of capital
  – Divide the capital accumulation equation by $K_t$
    
    $g_{Kt} = \frac{\Delta K_{t+1}}{K_t} = \frac{Y_t}{K_t} - \bar{d}$

• Plug the results into
  
  $g_{yt} = g_{At} + \frac{1}{3}g_{Kt} + \frac{2}{3}g_{Lyt}$

  
  $g_{At} = \bar{z}L = \bar{g}$
  
  $g_{Kt}^* = g_{yt}^*$
  
  $g_{Lyt}^* = 0$

  
  $g_{yt}^* = \bar{g} + \frac{1}{3}g_{yt}^* + \frac{2}{3} \cdot 0$

• Solve for the growth rate of output
  
  $g_{yt}^* = \frac{3}{2} \bar{g} = \frac{3}{2} \bar{z} \bar{L}$

• For the long-run combined model, this equation pins down
  – The growth rate of output
  – The growth rate of output per person

• The growth rate in the number of workers is zero.
  – The number of workers is a constant fraction of the population.
  – We’ve assumed that the population itself is constant.

  
  Therefore: $g_{Lyt} = 0$

• The growth rate of output is even larger in the combined model than in the Romer model.

• Output is higher in this model because
  – Ideas have a direct and an indirect effect.
  – Increasing productivity raises output because
    • productivity has increased
    • higher productivity results in a higher capital stock.
Output per Person

- The equation for the capital stock can be solved for the capital-output ratio along a balanced growth path.

- The capital to output ratio is proportional to the investment rate along a balanced growth path.

\[
\frac{K^*_t}{Y^*_t} = \frac{\bar{s}}{g_y^* + \bar{d}}
\]

- This solution for the capital-output ratio can be substituted back into the production function and solved to get:

\[
y^*_t = \frac{Y^*_t}{L} = \left(\frac{\bar{s}}{g_y^* + \bar{d}}\right)^{1/2} A_t^{3/2} (1 - \bar{h})
\]

Transition Dynamics

- The Solow model and the combined model both have diminishing returns to capital.

- Thus, transition dynamics applies in both models.

- The principle of transition dynamics for the combined model
  - The farther below its balanced growth path an economy is, the faster the economy will grow.
  - The farther above its balanced growth path an economy is, the slower the economy will grow.

- A permanent increase in the investment rate in the combined model implies:
  - The balanced growth path of income is higher (parallel shift).
  - Current income is unchanged.
    - the economy is now below the new balanced growth path
  - The growth rate of income per capita is immediately higher.
    - the slope of the output path is steeper than the balanced growth path

- Growth in \( A_t \)
  - Leads to sustained growth in output per person along a balanced growth path

- Output \( y_t \)
  - Depends on the square root of the investment rate

- A higher investment rate
  - Raises the level of output per person along the balanced growth path.
• Changes in any parameter result in transition dynamics.

• The resulting theory:
  – Generates long-run growth through ideas
  – Explains differences in growth rates across countries through transition dynamics.