Coordinates, Now with More Dimensions....

2D Coordinates

Variables: x (indep), y (dep)

Axes: x-axis $\perp$ y-axis

Coordinates of a point: (a, b)

Go a units in x-direction, b in y.

Projection onto x-axis: $(a, 0)$

(drop a line $\perp$ from point to x-axis.)

Distance from origin: $\sqrt{a^2 + b^2}$

First quadrant:

All points (a, b) with $a \geq 0$, $b \geq 0$.

3D Coordinates

Variables: x, y (indep), z (dep)

Axes: x-axis $\perp$ y-axis $\perp$ z-axis

$\circlearrowleft$

(right hand rule)

Coordinates of a point: (a, b, c)

Go a units in x-dir, b in y, c in z.

Projection onto xy-plane: $(a, b, 0)$

(drop a line $\perp$ from point to xy-plane)

Distance from origin: $\sqrt{a^2 + b^2 + c^2}$

First **octant:**

All points (a, b, c) with $a, b, c \geq 0$

Continued Comparison

2D Equations

Lines:

$x = 2$ defines a vertical line.
All points $(2, b)$ for any b
$y = 3$ defines a horizontal line.
All points $(a, 3)$ for any a

Circles:

All points distance r from (h, k):

$$x^2 + y^2 = r^2$$

$$(x - h)^2 + (y - k)^2 = r^2$$

3D Equations

Planes:

$x = 2$ is a plane parallel to the yz-plane.
All points $(2, b, c)$ for any b, c.
$x = y$ defines a plane too.
All points (a, a, c) for any a, c.

Spheres:

All points distance r from (h, k, ℓ):

$$x^2 + y^2 + z^2 = r^2$$

$$(x - h)^2 + (y - k)^2 + (z - \ell)^2 = r^2$$

Vectors

Definition: A **vector** is a quantity that has both magnitude and direction, often represented by an arrow. We'll use either $\mathbf{v}$, $\vec{v}$, or $\vec{\mathbf{v}}$.

- Think of them as a generalization of a single number.
- They do not have a fixed position.
- Can have any number of dimensions.

What can we do with vectors?

We can add vectors.
Place the tail of $\vec{\mathbf{v}}$ at the head of $\vec{\mathbf{u}}$.
Define $\vec{\mathbf{u}} + \vec{\mathbf{v}}$ to be the vector starting
at the tail of $\vec{\mathbf{u}}$ and ending at the head of $\vec{\mathbf{v}}$.
Properties: $\vec{\mathbf{u}} + \vec{\mathbf{0}} = \vec{\mathbf{u}}$
$\vec{\mathbf{u}} + \vec{\mathbf{v}} = \vec{\mathbf{v}} + \vec{\mathbf{u}}$. (Parallelogram!)

We can stretch vectors
by a constant factor (a **scalar**).
Zero Properties:
$0 \cdot \vec{\mathbf{v}} = \vec{\mathbf{0}}$ and $c \cdot \vec{\mathbf{0}} = \vec{\mathbf{0}}$

So we can subtract vectors,
because $\vec{\mathbf{u}} - \vec{\mathbf{v}} = \vec{\mathbf{u}} + (-1)\vec{\mathbf{v}}$.
Distributive laws:
$(a + b) \cdot \vec{\mathbf{v}} = a \cdot \vec{\mathbf{v}} + b \cdot \vec{\mathbf{v}}$
$c \cdot (\vec{\mathbf{u}} + \vec{\mathbf{v}}) = c \cdot \vec{\mathbf{u}} + c \cdot \vec{\mathbf{v}}$

Grasping the magnitude of the situation

To write a vector in coordinates, place the tail at the origin and find the coordinates of the head.

- $\vec{\mathbf{u}} = \langle 1, 4, 3 \rangle$, then $|\vec{\mathbf{u}}| = \sqrt{1^2 + 4^2 + 3^2} = \sqrt{26}$.
- $\vec{\mathbf{v}} = \langle 0, -1, -3 \rangle$, then $|\vec{\mathbf{v}}| = \sqrt{0^2 + (-1)^2 + (-3)^2} = \sqrt{10}$.

Three special vectors: $\vec{\mathbf{i}} = \langle 1, 0, 0 \rangle$, $\vec{\mathbf{j}} = \langle 0, 1, 0 \rangle$, $\vec{\mathbf{k}} = \langle 0, 0, 1 \rangle$.

- Alternate form: $\vec{\mathbf{u}} = \vec{\mathbf{i}} + 4\vec{\mathbf{j}} + 3\vec{\mathbf{k}}$. **standard basis vectors**

We add vectors componentwise. And multiply scalars componentwise.

- $\vec{\mathbf{u}} + \vec{\mathbf{v}} = \langle 1 + 0, 4 + (-1), 3 + (-3) \rangle = \langle 1, 3, 0 \rangle$
- $\pi\vec{\mathbf{u}} = \pi\langle 1, 4, 3 \rangle = \langle \pi, 4\pi, 3\pi \rangle$ Stretch it!

Magnitude is a synonym for **length**, written $|\vec{\mathbf{v}}|$ or $\|\vec{\mathbf{v}}\|$.
You may need to find the **unit vector** in the same direction as $\vec{\mathbf{u}}$.

- Find the length of $\vec{\mathbf{u}}$ and divide by it!
- Example. Unit vector of $\langle 2, -1, -2 \rangle$ is

Vectors are the best way to understand Physics

Example. A 100 lb weight hangs from the ceiling. How much force is held by each rope?

Answer: The forces must be in equilibrium.
This means that the sum of all the forces equals $\vec{\mathbf{0}}$.

- Set up a coordinate system centered at the rope meeting place.
- Find the force vector on each rope.
  - The weight down gives $\vec{\mathbf{W}} = \langle 0, -100 \rangle$.
  - The first force $\vec{\mathbf{F}}_1 = F_1 \langle \underline{\hspace{3cm}} \rangle$ (magnitude · direction)
  - The second force $\vec{\mathbf{F}}_2 = F_2 \langle \underline{\hspace{3cm}} \rangle$
- Use equilibrium to get a system of equations, solve.

$$-\cos 50\, F_1 + \cos 32\, F_2 = 0 \quad \text{and} \quad \sin 50\, F_1 + \sin 32\, F_2 - 100 = 0$$

Solving gives $F_1 \approx 85$ lb and $F_2 \approx 65$ lb.