Coordinates, Now with More Dimensions....

2D Coordinates

Variables: x (indep), y (dep)

Axes: x-axis $\perp$ y-axis

3D Coordinates

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Coordinates of a point: (a, b)

Go a units in x-direction, b in y.

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$\circlearrowleft$

(right hand rule)

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⟲

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Continued Comparison

2D Equations	3D Equations
Lines: $x = 2$ defines	

Continued Comparison

2D Equations

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3D Equations

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Circles:

All points distance r from (h, k):

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2D Equations	**3D Equations**
Lines:	**Planes:**
$x = 2$ defines a vertical line.	$x = 2$ is a plane parallel to the yz-plane.
All points $(2, b)$ for any b	All points $(2, b, c)$ for any b, c.
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Circles:	**Spheres:**
All points distance r from (h, k):	All points distance r from (h, k, ℓ):
$x^2 + y^2 = r^2$	$x^2 + y^2 + z^2 = r^2$
$(x - h)^2 + (y - k)^2 = r^2$	$(x - h)^2 + (y - k)^2 + (z - \ell)^2 = r^2$

Vectors

Definition: A **vector** is a quantity that has both magnitude and direction, often represented by an arrow. We'll use either $\mathbf{v}$, $\vec{v}$, or $\vec{\mathbf{v}}$.

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- Can have any number of dimensions.

What can we do with vectors?

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by a constant factor (a **scalar**).

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Distributive laws:
$(a + b) \cdot \vec{\mathbf{v}} = a \cdot \vec{\mathbf{v}} + b \cdot \vec{\mathbf{v}}$
$c \cdot (\vec{\mathbf{u}} + \vec{\mathbf{v}}) = c \cdot \vec{\mathbf{u}} + c \cdot \vec{\mathbf{v}}$

Grasping the magnitude of the situation

To write a vector in coordinates, place the tail at the origin and find the coordinates of the head.

- $\vec{\mathbf{u}} = \langle 1, 4, 3 \rangle$,
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- $\vec{\mathbf{u}} = \langle 1, 4, 3\rangle$, then $|\vec{\mathbf{u}}| = \sqrt{1^2 + 4^2 + 3^2} = \sqrt{26}$.
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- Find the length of $\vec{\mathbf{u}}$ and divide by it!

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- Example. Unit vector of $\langle 2, -1, -2 \rangle$ is

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Example. A 100 lb weight hangs from the ceiling. How much force is held by each rope?

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- Set up a coordinate system centered at the rope meeting place.

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Answer: The forces must be in equilibrium.
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Solving gives $F_1 \approx 85$ lb and $F_2 \approx 65$ lb.