Lines, Planes, and Automobiles!

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Two Lines

In three dimensions, two lines can

- be parallel
- intersect
- be skew

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Example. Find the equation of the line that passes through $A = (2, 4, -3)$ and $B = (3, -1, 1)$.

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Answer: In other words, ________________________________.

Never the twain shall meet

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$$\text{Romeo} : \langle 1+t, -2+3t, 4-t \rangle$$

$$\text{Juliet} : \langle 2s, 3+s, -3+4s \rangle$$

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Alternate forms

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Example. What is the angle between the planes

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Use the equation of the line and the plane:
$2x + 3y - 5z + 10 = 0 \rightsquigarrow 2(1 + 2t) + 3(0 + 3t) - 5(-1 - 5t) + 10 = 0$

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$\vec{P}_1-\vec{P}_0=\frac{-17}{38}\langle 2,3,-5\rangle$, so

$|\vec{P}_1-\vec{P}_0|=\frac{17}{38}\sqrt{2^2+3^2+(-5)^2}=\frac{17}{\sqrt{38}}$.