Drawing simple 3-D surfaces

Definition: **Cylinders** are surfaces where all slices are the same.

Example. $z = x^2$. $\longleftarrow$ y is NOT in this equation; y can be anything.
For any choice of $y = k$ (parallel to ____-plane),
the surface looks like a parabola opening toward the positive z-axis.
It is a parabolic cylinder.

Example. $y^2 + z^2 = 1$. $\longleftarrow$ x is not in this equation.
For any choice of $x = k$, the surface looks like a unit circle.

Quadric surfaces

Definition: A **quadric surface** is defined by an equation of the form:

$$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + Gx + Hy + Iz + J = 0.$$

They are the analog of conic sections in two dimensions.
Through rotation or translation, we need only consider two types:

$$Ax^2 + By^2 + Cz^2 + J = 0 \quad \text{and} \quad Ax^2 + By^2 + Iz = 0.$$

Strategy: Take slices in each coordinate direction, piece the slices together to understand the surface.

$$\begin{cases} x = \text{constant } k \\ y = \text{constant } k \\ z = \text{constant } k \end{cases}$$

Example. $x^2 + \frac{y^2}{9} + \frac{z^2}{4} = 1.$

When $z = 0$, $x^2 + \frac{y^2}{9} = 1$ is an ellipse. $(-2 \leq k \leq 2)$

When $z = k$, $x^2 + \frac{y^2}{9} = 1 - \frac{k^2}{4}$ is an ellipse when $1 - \frac{k^2}{4} \geq 0$.

When $x = k$, $\frac{y^2}{9} + \frac{z^2}{4} = 1 - k^2$ is an ellipse

When $y = k$, $x^2 + \frac{z^2}{4} = 1 - \frac{k^2}{9}$ is an ellipse

Every slice is an ellipse $\rightsquigarrow$ surface is an ellipsoid.

Example. $z = y^2 - x^2$

Slices	$x = k$	$y = k$	$z = k$
Eqn Format	$z = y^2 - k^2$	$z = k^2 - x^2$	$k = y^2 - x^2$
Conic section			
Sketches			

Assemble together:

Need to know

- There are six different families of quadric surfaces.

Ellipsoid (Sphere) $+\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	Cone $+\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=0$
Elliptic paraboloid $\frac{z}{c}=\frac{x^2}{a^2}+\frac{y^2}{b^2}$	Hyperboloid of one sheet $+\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$
Hyperbolic paraboloid $\frac{z}{c}=\frac{x^2}{a^2}-\frac{y^2}{b^2}$	Hyperboloid of two sheets $-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

- Matching equations to surfaces.
- More variety than conic sections but same building blocks.
- How to find slices, assemble to a rough sketch.

Online Resources:

https://www.youtube.com/watch?v=LBiiOEiD3Yk

http://tutorial.math.lamar.edu/Classes/CalcIII/QuadricSurfaces.aspx