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Example. $y^2 + z^2 = 1$. $\longleftarrow$ x is not in this equation.
For any choice of $x = k$, the surface looks like a unit circle.

Quadric surfaces

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Ax^2

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Every slice is an ellipse $\rightsquigarrow$ surface is an ellipsoid.

Example. $z = y^2 - x^2$

Slices	$x = k$	$y = k$	$z = k$
Eqn Format	$z = y^2 - k^2$	$z = k^2 - x^2$	$k = y^2 - x^2$
Conic section			
Sketches			

Assemble together:

Need to know

- There are six different families of quadric surfaces.

Ellipsoid (Sphere) $$+\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$	Cone $$+\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=0$$
Elliptic paraboloid $$\frac{z}{c}=\frac{x^2}{a^2}+\frac{y^2}{b^2}$$	Hyperboloid of one sheet $$+\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$$
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- More variety than conic sections but same building blocks.

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- Matching equations to surfaces.
- More variety than conic sections but same building blocks.
- How to find slices, assemble to a rough sketch.

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