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Example. Show that if a particle moves with constant speed, then the velocity and acceleration vectors are orthogonal.

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In our example, $s = \sqrt{2}t$, so $t = \frac{s}{\sqrt{2}}$. Substituting,

$$\vec{\mathbf{r}}(s) = \cos \tfrac{s}{\sqrt{2}}\,\vec{\mathbf{i}} + \sin \tfrac{s}{\sqrt{2}}\,\vec{\mathbf{j}} + \tfrac{s}{\sqrt{2}}\,\vec{\mathbf{k}}.$$

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The circle that lies along the curve has radius $1/\kappa$. (**!**)

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Question: Should $\kappa(t)$ be a constant?

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Differentiate $\vec{\mathbf{v}}(t) = v(t)\vec{\mathbf{T}}(t)$. (magnitude (speed) times unit direction)

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How much of the acceleration is toward $\vec{\mathbf{T}}$, $\vec{\mathbf{N}}$, and $\vec{\mathbf{B}}$?

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So $a_T = v' = \frac{\vec{\mathbf{v}} \cdot \vec{\mathbf{a}}}{v}$

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Nice symmetry!

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Example. Find tang'l, normal comp's of acceleration for $\vec{\mathbf{r}} = \langle t, 2t, t^2 \rangle$.