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$f : x \mapsto f(x)$	$f : (x, y) \mapsto f(x, y) = z$
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  - A curve represents points in the domain at the same "height".

The domain of a function of two variables

Example. What is the domain of the functions

$$f(x,y) = \frac{\sqrt{x+y+1}}{x-1} \qquad \text{and} \qquad g(x,y) = x\ln(y^2 - x)?$$

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Example. Sketch the following functions

$$f(x, y) = 6 - 3x - 2y \qquad \text{and} \qquad g(x, y) = \sqrt{9 - x^2 - y^2}.$$

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Example. Sketch the level curves of the function $h(x, y) = \sqrt{9 - x^2 - y^2}$ for $k = 0, 1, 2, 3$.

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  This gives a *surface* on which the function has a constant value.

Think: Which positions in this room have the same temperature?