More partials

This works with more variables too.

$\frac{\partial}{\partial z}\left(e^{xy}\ \ln z\right) =$ ___________ and $\frac{\partial}{\partial x}\left(e^{xy}\ \ln z\right) =$ ___________

We can also take higher derivatives.

$$\frac{\partial}{\partial x}\frac{\partial}{\partial x} f(x, y) \quad \textbf{or} \quad \frac{\partial^2}{\partial x^2} f(x, y) \quad \textbf{or} \quad f_{xx}(x, y)$$

We might even decide to mix our partial derivatives.

$$f_{xy} = (f_x)_y = \frac{\partial}{\partial y}\frac{\partial}{\partial x} f(x, y).$$

A big deal: Partial Differential Equations

- Laplace's Equation: $\frac{\partial^2}{\partial x^2} u(x, y) + \frac{\partial^2}{\partial y^2} u(x, y) = 0$ is a PDE.
  - Solutions (fcns u that satisfy) give formulas related to distribution of heat on a surface, how fluids & electricity flow.
- Wave Equation: $\frac{\partial^2}{\partial t^2} u(x, t) = a \frac{\partial^2}{\partial x^2} u(x, t)$ is a PDE.
  - Solutions describe the position of waves as a function of time.

Clairaut's Theorem

Example. Calculate all second-order partial derivatives of

$$f(x, y) = x^3 + x^2y^3 - 2y^2.$$

$f_x =$	$f_y =$
$f_{xx} =$	$f_{yx} =$
$f_{xy} =$	$f_{yy} =$

Notice: ______________________________

Clairaut's Theorem (mid 1700's)
Suppose $f(x, y)$ is defined on a disk D containing (a, b).
If f_{xy} and f_{yx} are continuous on D, **then** $f_{xy}(a, b) = f_{yx}(a, b)$.

Consequence: Order partial derivatives however you want.

$$f_{xyzz} = f_{zxyz} = f_{zyzx} = \cdots$$

Interpretation of partial derivatives

Function of one variable

$\frac{d}{dx} f(x)$ at $x = a$

slope of tangent line to the curve

$$y = f(x)$$

at $x = a$.

"What is the rate of change of $f(x)$ as x changes?"

Function of several variables

$\frac{\partial}{\partial x} f(x, y)$ at $(x, y) = (a, b)$

slope of tangent line to the curve on the surface $z = f(x, y)$ where sliced by the vertical plane $y = b$ at $x = a$.

"If y is fixed, what is the rate of change of $f(x, y)$ as x changes?"