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This works with more variables too.

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A big deal: Partial Differential Equations

- Laplace's Equation: $\frac{\partial^2}{\partial x^2}u(x,y) + \frac{\partial^2}{\partial y^2}u(x,y) = 0$ is a PDE.

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    - Solutions describe the position of waves as a function of time.

Clairaut's Theorem

Example. Calculate all second-order partial derivatives of

$$f(x, y) = x^3 + x^2y^3 - 2y^2.$$

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Clairaut's Theorem (mid 1700's)
Suppose $f(x, y)$ is defined on a disk D containing (a, b).
If f_{xy} and f_{yx} are continuous on D, **then** $f_{xy}(a, b) = f_{yx}(a, b)$.

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$$f_{xyzz} = f_{zxyz} = f_{zyzx} = \cdots$$

Interpretation of partial derivatives

Function of one variable

$\frac{d}{dx}f(x)$ at $x = a$

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slope of tangent line to the curve

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Function of several variables

$\frac{\partial}{\partial x}f(x, y)$ at $(x, y) = (a, b)$

slope of tangent line to the curve on the surface $z = f(x, y)$ where sliced by the vertical plane $y = b$ at $x = a$.

Interpretation of partial derivatives

Function of one variable	**Function of several variables**
$\frac{d}{dx}f(x)$ at $x = a$	$\frac{\partial}{\partial x}f(x,y)$ at $(x,y) = (a,b)$
slope of tangent line to the curve $$y = f(x)$$ at $x = a$.	slope of tangent line to the curve on the surface $z = f(x,y)$ where sliced by the vertical plane $y = b$ at $x = a$.
"What is the rate of change of $f(x)$ as x changes?"	"If y is fixed, what is the rate of change of $f(x,y)$ as x changes?"