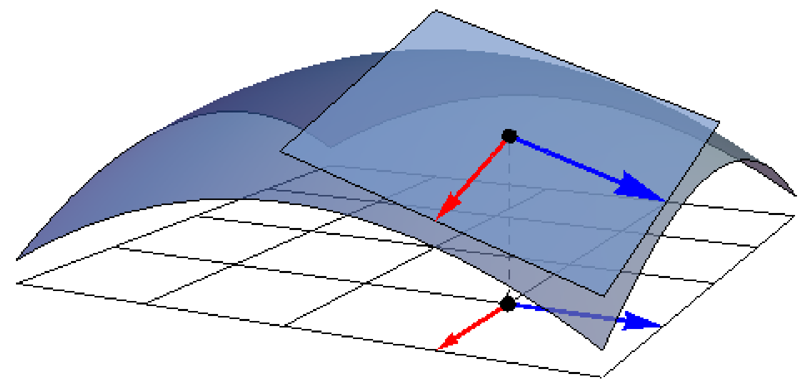
Tangenty

$\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at (x_0, y_0) are slopes of tangent lines along the surface.



They lie in the **tangent plane**.

"When we zoom into a smooth surface, the surface looks like a plane."

For any curve on the surface through $\left(x_0, y_0, f(x_0, y_0)\right)$, the tangent line to the curve would be in this plane too.

The equation of this tangent plane is easy.

$$(z - z_0) = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

(Point-slope format)

Planey

Example. What is the eqn of the tangent plane to $z = xe^{xy}$ at $(1, 0)$?

Game Plan:

1. Find the partial derivatives, evaluate at $(1, 0)$. (Find slopes)

2. Determine the point on the surface. (Find point)

3. Write down equation of plane. (Point-slope formula)

Linear Approximation

Key idea: Use tangent plane as linear approximation to the function.

$T(x, y)$ gives a "good enough" value for $f(x, y)$ near (x_0, y_0).

Example. Use a linear approximation of $f(x, y) = xe^{xy}$ to approximate $f(1.1, -0.1)$.

Answer: $(1.1, -0.1)$ is a point near __________.

The tangent plane there is $T(x, y) = x + y$.

So: $f(1.1, -0.1) \approx T(1.1, -0.1) =$ ____________.

Note: $f(1.1, -0.1) = 1.1e^{-0.11} \approx .985$.

In more dimensions: Suppose $f(\vec{\mathbf{v}}) = f(w, x, y, z)$.

A linear approximation near $\vec{\mathbf{v}}_0 = (w_0, x_0, y_0, z_0)$ would be

$$f(\vec{\mathbf{v}}) - f(\vec{\mathbf{v}}_0) \approx f_w(\vec{\mathbf{v}}_0)(w - w_0) + f_x(\vec{\mathbf{v}}_0)(x - x_0) + f_y(\vec{\mathbf{v}}_0)(y - y_0) + f_z(\vec{\mathbf{v}}_0)(z - z_0)$$

Differentials

Differentials are the other way to understand linear approximations. How much does z change as x and y change?

$$dz = \frac{\partial z}{\partial x}\,dx + \frac{\partial z}{\partial y}\,dy$$

dz is a *approximation* for how much z actually changes.

Example. If $z = x^2 + 3xy - y^2$, find dz.

$dz =$

Conclusion: If x changes from $2 \to 2.05$, $dx =$ ______

If y changes from $3 \to 2.96$, $dy =$ ______.

We would expect z to change by __________.

The true change is .6449.