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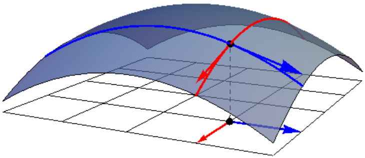

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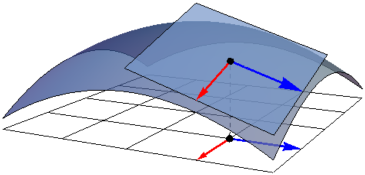

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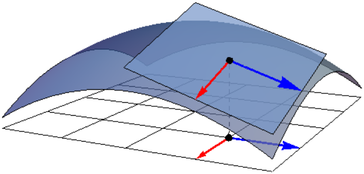

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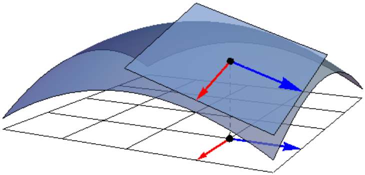

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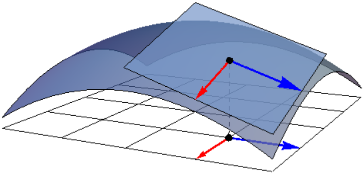

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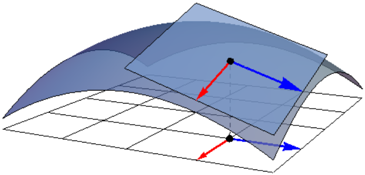

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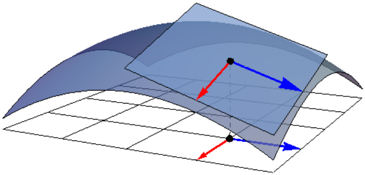

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(Point-slope format)

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A linear approximation near $\vec{\mathbf{v}}_0 = (w_0, x_0, y_0, z_0)$ would be

$$f(\vec{\mathbf{v}})-f(\vec{\mathbf{v}}_0) \approx f_w(\vec{\mathbf{v}}_0)(w-w_0)+f_x(\vec{\mathbf{v}}_0)(x-x_0)+f_y(\vec{\mathbf{v}}_0)(y-y_0)+f_z(\vec{\mathbf{v}}_0)(z-z_0)$$

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