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4. If $D = 0$, the test is inconclusive.

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This problem must have an absolute maximum, which must occur at a critical point. (Why?) Therefore $(x, y, z) = (2, 2, 1)$ is the absolute maximum, and the maximum volume is $xyz = 4$.