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Plot it to see the shape.

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First: Draw a picture!

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$$L = \lim_{n \to \infty} \sum_{i=1}^{n} \Delta L_i = \lim_{n \to \infty} \sum_{i=1}^{n} \sqrt{(f'(t_i^*))^2 + (g'(t_i^{**}))^2} \Delta t_i$$
(Similar to a Riemann Sum)

$$L = \int_{t=\alpha}^{t=\beta} \sqrt{\left[f'(t)\right]^2 + \left[g'(t)\right]^2} dt$$

Example. Find the arc length for the parametric curve  $\begin{cases} x = \sin 2t \\ y = \cos 2t \end{cases}$  for  $0 \le t \le 2\pi$ . What do we expect? What is this curve?

To find the arc length of a parametric curve, think  $L = \int dL$ .

How much arc length  $\Delta L$  does the curve traverse in one time unit  $\Delta t$ ?

$$\Delta L_i = \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2} = \sqrt{(f'(t_i^*)\Delta t_i)^2 + (g'(t_i^{**})\Delta t_i)^2}$$
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 $\begin{cases} x = \sin 2t \\ \text{Example.} \end{cases}$  Find the arc length for the parametric curve  $\begin{cases} y = \cos 2t \\ \text{for } 0 \leq t \leq 2\pi. \end{cases}$  What do we expect? What is this curve?

$$\int_{t=0}^{t=2\pi} \sqrt{[f'(t)]^2 + [g'(t)]^2} dt =$$

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... take derivatives & do the algebra ...

$$L = \int_{\theta=\alpha}^{\theta=\beta} \sqrt{\left[f(\theta)\right]^2 + \left[\frac{d}{d\theta}(f(\theta))\right]^2} d\theta$$

We can also understand this as  $dL = \sqrt{(r d\theta)^2 + (dr)^2}$