MATH 245, Spring 2015
Homework 3
due 10:45am on Wednesday, April 1.

Background reading: Sections 1.5, 5.1, and 5.3A.

- About Markov chains:
  http://www.sosmath.com/matrix/markov/markov.html
- About Random Walks:
  http://www.ceid.upatras.gr/courses/probweb/lectures/lecture26.ps

I ask that you do not contact previous Math Modeling students when completing this assignment. Provide details of calculations and assertions that you include. Don't forget to provide acknowledgments for those who helped you with the assignment and those resources that you consulted.

4-1. (5 pts) Consider two events E_1 and E_2 that are **dependent** events.

Will $\mathbb{P}(E_1$ **and** $E_2) \geq \mathbb{P}(E_1)\mathbb{P}(E_2)$ **OR** will $\mathbb{P}(E_1$ **and** $E_2) \leq \mathbb{P}(E_1)\mathbb{P}(E_2)$?

Determine all that you can.

[*You may wish to consider multiple cases, perhaps investigating using Venn diagrams.*]

4-2. (8 pts) Suppose you decide to visit Las Vegas (or Atlantic City, your choice) with exactly \$255 to spend. You decide to spend your time at the roulette wheel and bet either on red or black. As you may or may not know, a single-zero roulette wheel has eighteen red number spaces, eighteen black number spaces, and one green zero space, each as likely as one another. When you bet on red (or black), you win the amount of your bet if the ball lands on one of the red (or black) spaces, and lose the amount of your bet if not. Note that because of the green space, you will lose money on average.

(a) One strategy for heading to the casino is to bet one dollar each of 255 times. What is the expected value of the amount of money you will have at the end of the day?
[*Be explicit about what you are assuming when you give your answer.*]

(b) Now consider this new strategy:

- In your first game, you bet one dollar on red. If you win, you stop playing for the day, and walk away having won a net one dollar.
- If you lose, you play a second game but this time bet two dollars on red. Again, if you win, you walk away having won one dollar.
- If you lose, you play again and this time bet four dollars. Continue in this fashion, walking away if you win at any step, or betting twice as much in each game following a loss, until you have no more money.

What is the expected value of the amount of money you will have at the end of the day?

(c) Do you prefer either of the strategies from part (a) or part (b), or would you spend your money in a different way?

4-3. (7 pts) Read the story below.

> In the Dark Ages, Harvard and Yale admitted only male students. Assume that, at that time, 80 percent of the sons of Harvard men went to Harvard and the rest went to Yale, and 60 percent of the sons of Yale men went to Yale and the rest went to Harvard.

(a) Set up a Markov Chain model to simulate this situation.

(b) Suppose that in the first generation, there are 100 students at each college, and that each man has one son. What would be the distribution of college students in the next generation?

(c) Determine the equilibrium distribution of Harvard and Yale students.