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  - Translate back to the real-world situation, see if it reflects reality.
  - Were the simplifying assumptions good? Based in reality?
  - An honest discussion about the errors inherent to your model.
  - Is the model a good model? (Criteria next class.)

Sidebar: The Mathematical definition of Error

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Most of the time, we discuss the absolute value of percentage error.

In other words, 5% error means the error is either -5% or 5%.

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Error: If layers are not parallel (off by $\alpha°$), the % errors can be large!

α	1	5	10	30
Model d	1.031	1.031	1.031	1.031
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Important: You can not avoid Formulation Errors.
Scrutinize and discuss all **explicit** and **implicit** assumptions.

Simplifying Assumptions

Problem Statement: Which products should Waldbaums feature at the endcap of Aisle 5 in order to maximize profit?

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Then T_1 might be ____ seconds or ____ seconds,
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Observation Errors in a model can be reduced by measuring many times and taking an average. (Polling averages, Nate Silver)

Computational Errors

3. **Truncation Errors** occur when you approximate an incalculable function.

Question: When is $x^5 + x - 1 = 0$? What is $\sin 1$? Numerically?

Answer: Use a Taylor series approximation:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots .$$

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$1.2300001 \cdot 1.2300001 = 1.5129002,\ 1.2300001 \cdot 1.5129002 = 1.8608674,$

$1.2300001^{10} = \mathbf{7.9259523}$

True answer: $7.925952539912863452584748018737649320039805 \cdots$

Descriptively Realistic

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> *The date of the next full moon is 29 days after the date of the last full moon.*

Is this model descriptively realistic? _____ Why?