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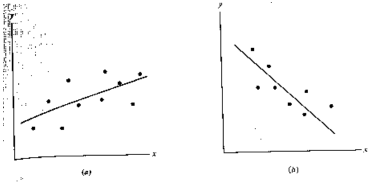
What can we conclude when two variables are highly **correlated**?



Positive Correlation

High values of x are associated with high values of y.

Negative Correlation

High values of x are associated with low values of y.

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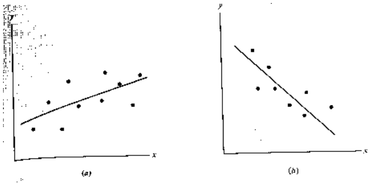

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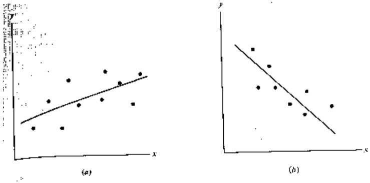

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High values of x are associated with high values of y.	High values of x are associated with low values of y.

The **correlation coefficient,** R^2 is a number between 0 and 1.

Values near 1 show strong correlation (data lies almost on a line).

Values near 0 show weak correlation (data doesn't lie on a line).

Calculating the R^2 Statistic

To find R^2, you need data and its best fit *linear* regression.

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- The **error sum of squares**: $SSE = \sum_i \left[y_i - f(x_i)\right]^2$.
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Is my R^2 good? Use a critical value table for R. (Note: not R^2.)
`http://www.gifted.uconn.edu/siegle/research/correlation/corrchrt.htm`

Calculating the R^2 Statistic

Example. (cont'd from notes p. 33) What is R^2 for the data set:
$\{(1.0, 3.6), (2.1, 2.9), (3.5, 2.2), (4.0, 1.7)\}$?

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$$= (.0017)^2 + (-0.033)^2 + (0.114)^2 + (-0.083)^2 = 0.0210$$

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$$= (1)^2 + (0.3)^2 + (-0.4)^2 + (-0.9)^2 = 2.06$$

- Now calculate $R^2 = 1 - \frac{SSE}{SST} = 1 - \frac{0.0210}{2.06} = 1 - .01 = 0.99$.

Another R^2 Calculation

Example. Estimating weight from height.

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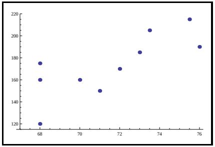

ht.	wt.
68	160
70	160
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72	170

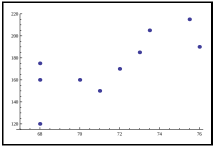
Another R^2 Calculation

Example. Estimating weight from height.

Here is a list of heights and weights for ten students.

We calculate the line of best fit:

$$(\text{weight}) = 7.07(\text{height}) - 333.$$



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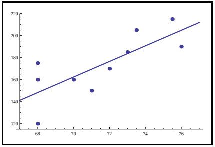

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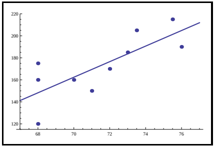

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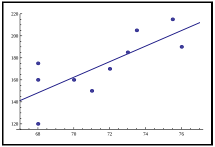

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Now find the correlation coefficient: $(\overline{w} = 173)$

$SSE = \sum_{i=1}^{10} \left[w_i - (7.07\, h_i - 333)\right]^2 \approx 2808$

$SST = \sum_{i=1}^{10} [w_i - 173]^2 = 6910$

So $R^2 = 1 - (2808/6910) = 0.59$

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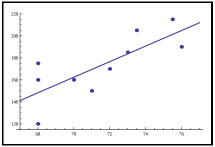

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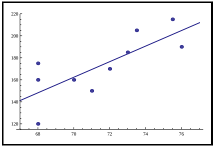

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We can introduce another variable to see if the fit improves.

Multiple Linear Regression

Add waist measurements to the data!

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68	34	160
70	32	160
71	31	150
68	29	120
68	34	175
76	34	190
73.5	38	205
75.5	34	215
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We wish to calculate a *linear* relationship such as:

$$(\text{weight}) = a\,(\text{height}) + b\,(\text{waist}) + c.$$

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Do a regression to find the *best-fit plane*:

Use the least-squares criterion. Minimize:

$$SSE = \sum_{(h_i, ws_i, wt_i)} \left[wt_i - (a \cdot h_i + b \cdot ws_i + c) \right]^2.$$

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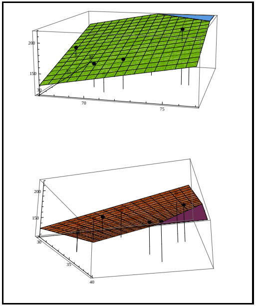
This finds that the best fit plane is (coeff sign)

$$(\text{weight}) = 4.59(\text{height}) + 6.35(\text{waist}) - 368.$$

Multiple Linear Regression

Visually, we might expect a plane to do a better job fitting the points than the line.



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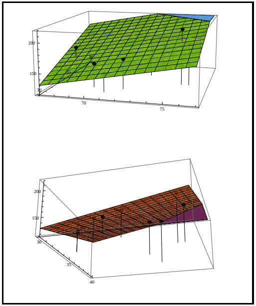
- Now calculate R^2.

Calculate $SSE = \sum_{i=1}^{10}(w_i - f(h_i, ws_i))^2 \approx 955$

SST does not change: (why?)

$\sum_{i=1}^{10}(w_i - 173)^2 = 6910$



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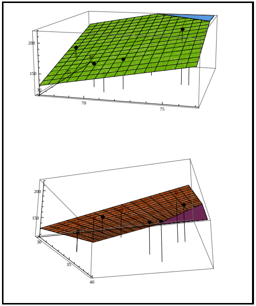

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So $R^2 = 1 - (955/6910) = 0.86$, an excellent correlation.

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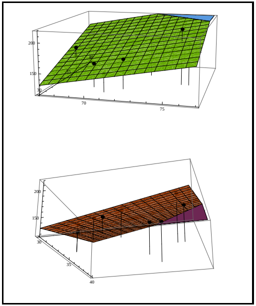

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- When you introduce more variables, SSE can only go down, so R^2 always increases.

Notes about the Correlation Coefficient

Example. Time and Distance (pp. 190)
Data collected to predict driving time from home to school.

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- R^2 increases as the number of variables increase.
- This doesn't mean that the fit is better!

Notes about the Correlation Coefficient

Example. Cancer and Fluoridation. (pp. 188–189)

Does fluoride in the water cause cancer?

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T = log of years of fluoridation

C = cancer mortality rate

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C = cancer mortality rate

Use a linear regression to find that
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Notes about the Correlation Coefficient

Example. Cancer and Fluoridation. (pp. 188–189)

Does fluoride in the water cause cancer?

Variables:

T = log of years of fluoridation A = % of population over 65.

C = cancer mortality rate

Use a linear regression to find that

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Notes about the Correlation Coefficient

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T = log of years of fluoridation A = % of population over 65.

C = cancer mortality rate

Use a linear regression to find that

$C = \mathbf{27.1}\,T + 181$, with an $R^2 = 0.047$.

Compare to a multiple linear regression of

$C = \mathbf{0.566}\,T + 10.6A + 85.8$, with an $R^2 = 0.493$.

- Be suspicious of a low R^2.

Notes about the Correlation Coefficient

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- **CAN** determine relative influence of one variable in two models.