Simulation Modeling

Goal: Use probabilistic methods to analyze deterministic and probabilistic models.

Example. Determine the best elevator delivery scheme.

- The wait is too long, too many stops along the way.

Simulation Modeling

Goal: Use probabilistic methods to analyze deterministic and probabilistic models.

Example. Determine the best elevator delivery scheme.

- The wait is too long, too many stops along the way.
- Inconvenient to experiment with alternate delivery schemes.
  - Disrupt normal service
  - Take surveys of customers
  - Confuse regular customers

Simulation Modeling

Goal: Use probabilistic methods to analyze deterministic and probabilistic models.

Example. Determine the best elevator delivery scheme.

- The wait is too long, too many stops along the way.
- Inconvenient to experiment with alternate delivery schemes.
  - Disrupt normal service
  - Take surveys of customers
  - Confuse regular customers
- Alternatively, run a computer **simulation**. Write a computer program that models the system of elevators, including:
  - Time of arrival of passengers (a random event)
  - Passenger destination (a random event)
  - Capacity of elevator (fixed by system)
  - Speed of elevator (fixed by system)
  - Current delivery scheme

Simulation Modeling

Once you have written the computer program,

Verify that the simulation models the current real-world situation

Then, modify various parameters in order to simulate a new delivery scheme.

Simulation Modeling

Once you have written the computer program,

Verify that the simulation models the current real-world situation

- Run the model many times.
- Have the computer keep track of data, such as average wait time, number of stops it takes, longest queue, etc.

Then, modify various parameters in order to simulate a new delivery scheme.

Simulation Modeling

Once you have written the computer program,

Verify that the simulation models the current real-world situation

- Run the model many times.
- Have the computer keep track of data, such as average wait time, number of stops it takes, longest queue, etc.

Then, modify various parameters in order to simulate a new delivery scheme.

- How do the data change?
- Is the alternate scheme better or worse?
- Determine how to implement to cause minimal disruption.

Monte Carlo Simulations

Definition: A simulation that incorporates an element of randomness is called a **Monte Carlo** simulation.

PROS:

CONS:

Monte Carlo Simulations

Definition: A simulation that incorporates an element of randomness is called a **Monte Carlo** simulation.

PROS:

- It is a relatively easy method to approximate complex systems.

CONS:

Monte Carlo Simulations

Definition: A simulation that incorporates an element of randomness is called a **Monte Carlo** simulation.

PROS:

- It is a relatively easy method to approximate complex systems.
- Once built, it allows for tinkering—easy to do sensitivity analysis.

CONS:

Monte Carlo Simulations

Definition: A simulation that incorporates an element of randomness is called a **Monte Carlo** simulation.

PROS:

- It is a relatively easy method to approximate complex systems.
- Once built, it allows for tinkering—easy to do sensitivity analysis.
- It can model systems over difficult-to-measure time frames.

CONS:

Monte Carlo Simulations

Definition: A simulation that incorporates an element of randomness is called a **Monte Carlo** simulation.

PROS:

- It is a relatively easy method to approximate complex systems.
- Once built, it allows for tinkering—easy to do sensitivity analysis.
- It can model systems over difficult-to-measure time frames.

CONS:

- You have to build it. (Expensive to develop!)
- Requires computing power and time.

Monte Carlo Simulations

Definition: A simulation that incorporates an element of randomness is called a **Monte Carlo** simulation.

PROS:

- It is a relatively easy method to approximate complex systems.
- Once built, it allows for tinkering—easy to do sensitivity analysis.
- It can model systems over difficult-to-measure time frames.

CONS:

- You have to build it. (Expensive to develop!)
- Requires computing power and time.
- Makes you over-confident in the results.

Monte Carlo Simulations

Definition: A simulation that incorporates an element of randomness is called a **Monte Carlo** simulation.

PROS:

- It is a relatively easy method to approximate complex systems.
- Once built, it allows for tinkering—easy to do sensitivity analysis.
- It can model systems over difficult-to-measure time frames.

CONS:

- You have to build it. (Expensive to develop!)
- Requires computing power and time.
- Makes you over-confident in the results.
- Dealing with probability, so results will always be of the form: "With 95% probability, the wait time will be less than 2 minutes."

Simulating flipping a coin

Example. Get a computer to simulate flipping a fair coin 20 times.

To simulate a random event, use one of the *Mathematica* commands:

- `RandomInteger` gives a pseudo-random *integer*.

- `RandomReal` gives a pseudo-random *real number*.

Simulating flipping a coin

Example. Get a computer to simulate flipping a fair coin 20 times.

To simulate a random event, use one of the *Mathematica* commands:

- `RandomInteger` gives a pseudo-random *integer*.
  - `RandomInteger[]` (no input) gives either 0 or 1.
  - `RandomInteger[5]` gives an integer from 0 to 5.
  - `RandomInteger[{1, 10}]` gives an integer from 1 to 10.

- `RandomReal` gives a pseudo-random *real number*.

Simulating flipping a coin

Example. Get a computer to simulate flipping a fair coin 20 times.

To simulate a random event, use one of the *Mathematica* commands:

- `RandomInteger` gives a pseudo-random *integer*.
  - `RandomInteger[]` (no input) gives either 0 or 1.
  - `RandomInteger[5]` gives an integer from 0 to 5.
  - `RandomInteger[{1, 10}]` gives an integer from 1 to 10.
  - `RandomInteger[{1, 10},20]` gives a list of 20 such integers.
- `RandomReal` gives a pseudo-random *real number*.

The first input gives the range; a second input tells how many to make.

Simulating flipping a coin

Example. Get a computer to simulate flipping a fair coin 20 times.

To simulate a random event, use one of the *Mathematica* commands:

- `RandomInteger` gives a pseudo-random *integer*.
  - `RandomInteger[]` (no input) gives either 0 or 1.
  - `RandomInteger[5]` gives an integer from 0 to 5.
  - `RandomInteger[{1, 10}]` gives an integer from 1 to 10.
  - `RandomInteger[{1, 10},20]` gives a list of 20 such integers.
- `RandomReal` gives a pseudo-random *real number*.
  - `RandomReal[]` (no input) gives a real number between 0 or 1.
  - `RandomReal[{0.1, 0.2}]` gives a real number from 0.1 to 0.2.

The first input gives the range; a second input tells how many to make.

Simulating flipping a coin

Example. Get a computer to simulate flipping a fair coin 20 times.

To simulate a random event, use one of the *Mathematica* commands:

- `RandomInteger` gives a pseudo-random *integer*.
  - `RandomInteger[]` (no input) gives either 0 or 1.
  - `RandomInteger[5]` gives an integer from 0 to 5.
  - `RandomInteger[{1, 10}]` gives an integer from 1 to 10.
  - `RandomInteger[{1, 10},20]` gives a list of 20 such integers.
- `RandomReal` gives a pseudo-random *real number*.
  - `RandomReal[]` (no input) gives a real number between 0 or 1.
  - `RandomReal[{0.1, 0.2}]` gives a real number from 0.1 to 0.2.
  - `RandomReal[{0.1, 0.2},15]` gives a list of 15 such numbers.

The first input gives the range; a second input tells how many to make.

Simulating flipping a coin

Example. Get a computer to simulate flipping a fair coin 20 times.

To simulate a random event, use one of the *Mathematica* commands:

- `RandomInteger` gives a pseudo-random *integer*.
  - `RandomInteger[]` (no input) gives either 0 or 1.
  - `RandomInteger[5]` gives an integer from 0 to 5.
  - `RandomInteger[{1, 10}]` gives an integer from 1 to 10.
  - `RandomInteger[{1, 10},20]` gives a list of 20 such integers.
- `RandomReal` gives a pseudo-random *real number*.
  - `RandomReal[]` (no input) gives a real number between 0 or 1.
  - `RandomReal[{0.1, 0.2}]` gives a real number from 0.1 to 0.2.
  - `RandomReal[{0.1, 0.2},15]` gives a list of 15 such numbers.

The first input gives the range; a second input tells how many to make.

The numbers produced by a random number generator are never truly random because they are produced by an algorithm on a deterministic machine.

Simulating flipping a coin

Example. Get a computer to simulate flipping a fair coin 20 times.

Let's choose a convention: 1='Head' and 0='Tail'.

Simulating flipping a coin

Example. Get a computer to simulate flipping a fair coin 20 times.

Let's choose a convention: 1=‘Head’ and 0=‘Tail’.
Evaluating `RandomInteger[1,20]` generates a list of 20 coin tosses.

Simulating flipping a coin

Example. Get a computer to simulate flipping a fair coin 20 times.

Let's choose a convention: 1=‘Head’ and 0=‘Tail’.
Evaluating `RandomInteger[1,20]` generates a list of 20 coin tosses.

In[1]: `CoinFlips = RandomInteger[1,20]`
Out[1]: {1, 0, 1, 0, 1, 1, 0, 0, 1, 1, 1, 1, 1, 0, 0, 0, 1, 1, 1, 1}

Simulating flipping a coin

Example. Get a computer to simulate flipping a fair coin 20 times.

Let's choose a convention: 1='Head' and 0='Tail'.
Evaluating `RandomInteger[1,20]` generates a list of 20 coin tosses.

In[1]: `CoinFlips = RandomInteger[1,20]`
Out[1]: {1, 0, 1, 0, 1, 1, 0, 0, 1, 1, 1, 1, 1, 0, 0, 0, 1, 1, 1, 1}

The sum of this list is the total number of heads tossed.

In[2]: `Total[CoinFlips]`
Out[2]: 13

Simulating flipping a coin

Example. Get a computer to simulate flipping a fair coin 20 times.

Let's choose a convention: 1=‘Head’ and 0=‘Tail’.
Evaluating `RandomInteger[1,20]` generates a list of 20 coin tosses.

In[1]: `CoinFlips = RandomInteger[1,20]`
Out[1]: {1, 0, 1, 0, 1, 1, 0, 0, 1, 1, 1, 1, 1, 0, 0, 0, 1, 1, 1, 1}

The sum of this list is the total number of heads tossed.

In[2]: `Total[CoinFlips]`
Out[2]: 13

Running the commands again will simulate another trial of 20 flips.

If statements and For loops

In order to incorporate more complex aspects into the model, use If statements and For loops.

```
If[condition,t,f]
```

`If` statements and `For` loops

In order to incorporate more complex aspects into the model, use `If` statements and `For` loops.

`If[condition,t,f]`

- First, *Mathematica* evaluates the '`condition`'.
- If '`condition`' is true, the statement evaluates the '`t`' part.
- If '`condition`' is false, the statement evaluates the '`f`' part.

`If` statements and `For` loops

In order to incorporate more complex aspects into the model, use `If` statements and `For` loops.

`If[condition,t,f]`

- First, *Mathematica* evaluates the ‘`condition`’.
- If ‘`condition`’ is true, the statement evaluates the ‘`t`’ part.
- If ‘`condition`’ is false, the statement evaluates the ‘`f`’ part.

Examples of conditions:

`x<0` `(x==0) && (y!=1)` `RandomInteger[]==1`

Note the **double equals** sign `==` and **not equals** `!=`.

`If` statements and `For` loops

In order to incorporate more complex aspects into the model, use `If` statements and `For` loops.

`If[condition,t,f]`

- First, *Mathematica* evaluates the '`condition`'.
- If '`condition`' is true, the statement evaluates the '`t`' part.
- If '`condition`' is false, the statement evaluates the '`f`' part.

Examples of conditions:

`x<0`   `(x==0) && (y!=1)`   `RandomInteger[]==1`

Note the **double equals** sign `==` and **not equals** `!=`.

Examples.

- `If[x<0, -x, x]` is ____________________.

If statements and For loops

In order to incorporate more complex aspects into the model, use If statements and For loops.

`If[condition,t,f]`

- First, *Mathematica* evaluates the ‘`condition`’.
- If ‘`condition`’ is true, the statement evaluates the ‘`t`’ part.
- If ‘`condition`’ is false, the statement evaluates the ‘`f`’ part.

Examples of conditions:

`x<0` `(x==0) && (y!=1)` `RandomInteger[]==1`

Note the **double equals** sign == and **not equals** `!=`.

Examples.

- `If[x<0, -x, x]` is ____________________.
- `If[RandomInteger[] == 1, "Head", "Tail"]`:

Using `If` statements in `Table` commands

Goal: Model something that happens 7.5% of the time.

Call `RandomReal[]` to output: ______________________________.

Using `If` statements in `Table` commands

Goal: Model something that happens 7.5% of the time.

Call `RandomReal[]` to output: ______________________________.

The output satisfies ______________________ 7.5% of the time.

Anything to the left of the split will be taken as success.

Using `If` statements in `Table` commands

Goal: Model something that happens 7.5% of the time.

Call `RandomReal[]` to output: ______________________.

The output satisfies ______________________ 7.5% of the time.

Anything to the left of the split will be taken as success.

To model this is *Mathematica*, use an `If` statement.

```
trial = RandomReal[]
success = If[trial <= 0.075, 1, 0]
```

Using `If` statements in `Table` commands

Goal: Model something that happens 7.5% of the time.

Call `RandomReal[]` to output: ______________________.

The output satisfies ________________ 7.5% of the time.

Anything to the left of the split will be taken as success.

To model this is *Mathematica*, use an `If` statement.

```
trial = RandomReal[]
success = If[trial <= 0.075, 1, 0]
```

Alternatively, do this is one step:

```
If[RandomReal[] <= 0.075, 1, 0]
```

Using `If` statements in `Table` commands

That was: `If[RandomReal[] <= 0.075, 1, 0]`

Let's run this command many times and visualize the results: Remember that `Table` will repeat a command multiple times:

```
trials = Table[If[RandomReal[] <= 0.075, 1, 0], {500}];
```

Using If statements in Table commands

That was: `If[RandomReal[] <= 0.075, 1, 0]`

Let's run this command many times and visualize the results:
Remember that `Table` will repeat a command multiple times:

```
trials = Table[If[RandomReal[] <= 0.075, 1, 0], {500}];
```

Output: 500-entry list, where each entry is 0 (failure) or 1 (success).

Using If statements in Table commands

That was: `If[RandomReal[] <= 0.075, 1, 0]`

Let's run this command many times and visualize the results:
Remember that `Table` will repeat a command multiple times:

```
trials = Table[If[RandomReal[] <= 0.075, 1, 0], {500}];
```

Output: 500-entry list, where each entry is 0 (failure) or 1 (success).

Question: How many successes? (Expected value: $500 \cdot 0.075 = 37.5$)

Using `If` statements in `Table` commands

That was: `If[RandomReal[] <= 0.075, 1, 0]`

Let's run this command many times and visualize the results:
Remember that `Table` will repeat a command multiple times:

```
trials = Table[If[RandomReal[] <= 0.075, 1, 0], {500}];
```

Output: 500-entry list, where each entry is 0 (failure) or 1 (success).

Question: How many successes? (Expected value: $500 \cdot 0.075 = 37.5$)

- If we add the entries `Total[trials]`, we get # successes.
  One time I ran it had 32 successes.

Using `If` statements in `Table` commands

That was: `If[RandomReal[] <= 0.075, 1, 0]`

Let's run this command many times and visualize the results:
Remember that `Table` will repeat a command multiple times:

```
trials = Table[If[RandomReal[] <= 0.075, 1, 0], {500}];
```

Output: 500-entry list, where each entry is 0 (failure) or 1 (success).

Question: How many successes? (Expected value: $500 \cdot 0.075 = 37.5$)

- If we add the entries `Total[trials]`, we get # successes.
  One time I ran it had 32 successes.
- Alternatively, `Tally[trials]` gives how many times distinct entries appear. Output: `{{0, 468}, {1, 32}}`

Using If statements in Table commands

That was: `If[RandomReal[] <= 0.075, 1, 0]`

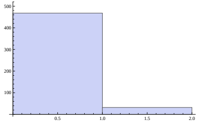
Let's run this command many times and visualize the results:
Remember that `Table` will repeat a command multiple times:

```
trials = Table[If[RandomReal[] <= 0.075, 1, 0], {500}];
```

Output: 500-entry list, where each entry is 0 (failure) or 1 (success).

Question: How many successes? (Expected value: $500 \cdot 0.075 = 37.5$)

- If we add the entries `Total[trials]`, we get # successes. One time I ran it had 32 successes.
- Alternatively, `Tally[trials]` gives how many times distinct entries appear. Output: `{{0, 468}, {1, 32}}`
- Last, we might want a visualization; Use `Histogram[trials]` to get:



If statements and For loops

```
For[start,test,incr,body]
```

If statements and For loops

`For[start,test,incr,body]`

- First, *Mathematica* evaluates the code in `start`.
- As long as `test` is true, (Can happen many times!)
- Continue to evaluate `body` and do the increment `incr`.

If statements and For loops

`For[start,test,incr,body]`

- First, *Mathematica* evaluates the code in `start`.
- As long as `test` is true, (Can happen many times!)
- Continue to evaluate `body` and do the increment `incr`.

Example. `For[i = 0, i < 4, i++, Print[i]]`

- First, *Mathematica* defines `i` to be equal to 0.
- Next, it checks to see if `i` is less than 4.
- It is, so it evaluates `Print[i]`, and increments `i` by 1 (`i++`).

If statements and For loops

`For[start,test,incr,body]`

- First, *Mathematica* evaluates the code in `start`.
- As long as `test` is true, (Can happen many times!)
- Continue to evaluate `body` and do the increment `incr`.

Example. `For[i = 0, i < 4, i++, Print[i]]`

- First, *Mathematica* defines `i` to be equal to 0.
- Next, it checks to see if `i` is less than 4.
- It is, so it evaluates `Print[i]`, and increments `i` by 1 (`i++`).
- Now `i` $= 1$, which is still < 4. So '`Print[i]`' is evaluated and `i` is incremented. Similarly for `i` $= 2$ and `i` $= 3$. Now `i` is incremented to 4, which is NOT < 4, and the loop terminates.

If statements and For loops

`For[start,test,incr,body]`

- First, *Mathematica* evaluates the code in `start`.
- As long as `test` is true, (Can happen many times!)
- Continue to evaluate `body` and do the increment `incr`.

Example. `For[i = 0, i < 4, i++, Print[i]]`

- First, *Mathematica* defines `i` to be equal to 0.
- Next, it checks to see if `i` is less than 4.
- It is, so it evaluates `Print[i]`, and increments `i` by 1 (`i++`).
- Now `i` $= 1$, which is still < 4. So '`Print[i]`' is evaluated and `i` is incremented. Similarly for `i` $= 2$ and `i` $= 3$. Now `i` is incremented to 4, which is NOT < 4, and the loop terminates.

This variable `i` is called a **counter**.
Be careful to name counters wisely! They are defined as variables.

Simulating flipping a coin

Example. Simulate flipping a fair coin 20 times using a `for` loop.

We'll write some **pseudocode**—words that explain what we want the computer to do, but won't actually work if we typed them in.

Simulating flipping a coin

Example. Simulate flipping a fair coin 20 times using a `for` loop.

We'll write some **pseudocode**—words that explain what we want the computer to do, but won't actually work if we typed them in.

- Run the loop 20 times.
  (Keep track using a counter: let `loopCount` vary from 1 to 20.)
- Each time the loop evaluates,
  - Generate a random integer between 0 and 1.
  - If '1' output 'Head', if '0', output 'Tail'.

Simulating flipping a coin

Example. Simulate flipping a fair coin 20 times using a `for` loop.

We'll write some **pseudocode**—words that explain what we want the computer to do, but won't actually work if we typed them in.

- Run the loop 20 times.
  (Keep track using a counter: let `loopCount` vary from 1 to 20.)
- Each time the loop evaluates,
  - Generate a random integer between 0 and 1.
  - If '1' output 'Head', if '0', output 'Tail'.

```
For[loopCount = 1, loopCount <= 20, loopCount++,
   flip = RandomInteger[];
   If[flip == 1, Print["Head"], Print["Tail"]]]
```

Simulating flipping a coin

Example. Simulate flipping a fair coin 20 times using a `for` loop.

We'll write some **pseudocode**—words that explain what we want the computer to do, but won't actually work if we typed them in.

- Run the loop 20 times.
  (Keep track using a counter: let `loopCount` vary from 1 to 20.)
- Each time the loop evaluates,
  - Generate a random integer between 0 and 1.
  - If '1' output 'Head', if '0', output 'Tail'.

```
For[loopCount = 1, loopCount <= 20, loopCount++,
   flip = RandomInteger[];
   If[flip == 1, Print["Head"], Print["Tail"]]]
```

- Notice the == and also the ; that separates the commands.

Simulating flipping a coin

Example. Simulate flipping a fair coin 20 times using a `for` loop.

We'll write some **pseudocode**—words that explain what we want the computer to do, but won't actually work if we typed them in.

- Run the loop 20 times.
  (Keep track using a counter: let `loopCount` vary from 1 to 20.)
- Each time the loop evaluates,
  - Generate a random integer between 0 and 1.
  - If '1' output 'Head', if '0', output 'Tail'.

```
For[loopCount = 1, loopCount <= 20, loopCount++,
   flip = RandomInteger[];
   If[flip == 1, Print["Head"], Print["Tail"]]]
```

- Notice the == and also the ; that separates the commands.
- `loopCount` is ONLY a counter; it does not change each step's evaluation.

Simulating flipping a coin

Pimp my code! Let's keep track of # heads and tails with counters.

`headCount`: # of heads so far. `tailCount`: # of tails so far.

Simulating flipping a coin

Pimp my code! Let's keep track of # heads and tails with counters.

`headCount`: # of heads so far. `tailCount`: # of tails so far.

- Zero out the counters: '`headCount`=0' and '`tailCount`=0'.
- Run the loop 20 times by having `loopCount` vary from 1 to 20.
- Each time the loop evaluates,
  - Generate a random integer between 0 and 1.
  - If '1', output 'Head' **AND**
  - If '0', output 'Tail' **AND**

Simulating flipping a coin

Pimp my code! Let's keep track of # heads and tails with counters.

`headCount`: # of heads so far. `tailCount`: # of tails so far.

- Zero out the counters: '`headCount`=0' and '`tailCount`=0'.
- Run the loop 20 times by having `loopCount` vary from 1 to 20.
- Each time the loop evaluates,
  - Generate a random integer between 0 and 1.
  - If '1', output 'Head' **AND** increase '`headCount`',
  - If '0', output 'Tail' **AND** increase '`tailCount`'.

Simulating flipping a coin

Pimp my code! Let's keep track of # heads and tails with counters.

`headCount`: # of heads so far. `tailCount`: # of tails so far.

- Zero out the counters: '`headCount`=0' and '`tailCount`=0'.
- Run the loop 20 times by having `loopCount` vary from 1 to 20.
- Each time the loop evaluates,
  - Generate a random integer between 0 and 1.
  - If '1', output 'Head' **AND** increase '`headCount`',
  - If '0', output 'Tail' **AND** increase '`tailCount`'.
- After 20 iterations, display '`headCount`' and '`tailCount`'.

Simulating flipping a coin

Pimp my code! Let's keep track of # heads and tails with counters.

headCount: # of heads so far. tailCount: # of tails so far.

- Zero out the counters: 'headCount=0' and 'tailCount=0'.
- Run the loop 20 times by having loopCount vary from 1 to 20.
- Each time the loop evaluates,
  - Generate a random integer between 0 and 1.
  - If '1', output 'Head' **AND** increase 'headCount',
  - If '0', output 'Tail' **AND** increase 'tailCount'.
- After 20 iterations, display 'headCount' and 'tailCount'.

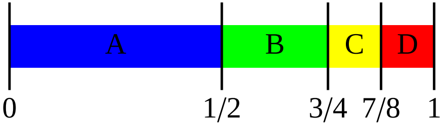
```
headCount=0; tailCount=0;
For[loopCount = 1, loopCount <= 20, loopCount++,
  If[RandomInteger[]==1,
     Print["Head"]; headCount++,          ← Notice the ';'
     Print["Tail"]; tailCount++]]         ← Notice the '++'
{headCount, tailCount}
```

Simulating rolling a biased die

Suppose you have a four-sided die, where the four sides (A, B, C, and D) come up with probabilities 1/2, 1/4, 1/8, and 1/8, respectively.

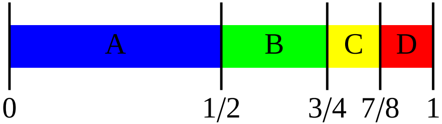
Simulating rolling a biased die

Suppose you have a four-sided die, where the four sides (A, B, C, and D) come up with probabilities 1/2, 1/4, 1/8, and 1/8, respectively.



Simulating rolling a biased die

Suppose you have a four-sided die, where the four sides (A, B, C, and D) come up with probabilities 1/2, 1/4, 1/8, and 1/8, respectively.



- Reset the counters: 'aCount=bCount=cCount=dCount=0'.
- For loopCount from 1 to 20,
  - Generate a random real number between 0 and 1.
  - If between 0 and 1/2, then output 'A' and aCount++
    if between 1/2 and 3/4, then output 'B' and bCount++
    if between 3/4 and 7/8, then output 'C' and cCount++
    if between 7/8 and 1, then output 'D' and dCount++
- Display 'aCount', 'bCount', 'cCount', and 'dCount'.

Simulating rolling a biased die

```
aCount = 0; bCount = 0; cCount = 0; dCount = 0;
For[loopCount = 1, loopCount <= 20, loopCount++,
 roll=RandomReal[];
 If[ 0  <= roll < 1/2, Print["a"]; aCount++];
 If[1/2 <= roll < 3/4, Print["b"]; bCount++];
 If[3/4 <= roll < 7/8, Print["c"]; cCount++];
 If[7/8 <= roll <= 1 , Print["d"]; dCount++];]
distribution = {aCount, bCount, cCount, dCount}
```

Simulating rolling a biased die

```
aCount = 0; bCount = 0; cCount = 0; dCount = 0;
For[loopCount = 1, loopCount <= 20, loopCount++,
 roll=RandomReal[];
 If[ 0  <= roll < 1/2, Print["a"]; aCount++];
 If[1/2 <= roll < 3/4, Print["b"]; bCount++];
 If[3/4 <= roll < 7/8, Print["c"]; cCount++];
 If[7/8 <= roll <= 1 , Print["d"]; dCount++];]
distribution = {aCount, bCount, cCount, dCount}
```

- Sample output: (each on its own line)

  a, a, a, d, d, b, a, a, d, a, a, a, a, d, b, a, a, c, a, b $\{12, 3, 1, 4\}$

Simulating rolling a biased die

```
aCount = 0; bCount = 0; cCount = 0; dCount = 0;
For[loopCount = 1, loopCount <= 20, loopCount++,
 roll=RandomReal[];
 If[ 0  <= roll < 1/2, Print["a"]; aCount++];
 If[1/2 <= roll < 3/4, Print["b"]; bCount++];
 If[3/4 <= roll < 7/8, Print["c"]; cCount++];
 If[7/8 <= roll <= 1 , Print["d"]; dCount++];]
distribution = {aCount, bCount, cCount, dCount}
```

- Sample output: (each on its own line)
  a, a, a, d, d, b, a, a, d, a, a, a, a, d, b, a, a, c, a, b $\{12, 3, 1, 4\}$
- These `If` statements all have no "False" part. (; vs ,)

Simulating rolling a biased die

```
aCount = 0; bCount = 0; cCount = 0; dCount = 0;
For[loopCount = 1, loopCount <= 20, loopCount++,
 roll=RandomReal[];
 If[ 0  <= roll < 1/2, Print["a"]; aCount++];
 If[1/2 <= roll < 3/4, Print["b"]; bCount++];
 If[3/4 <= roll < 7/8, Print["c"]; cCount++];
 If[7/8 <= roll <= 1 , Print["d"]; dCount++];]
distribution = {aCount, bCount, cCount, dCount}
```

- Sample output: (each on its own line)
  a, a, a, d, d, b, a, a, d, a, a, a, a, d, b, a, a, c, a, b $\{12, 3, 1, 4\}$
- These `If` statements all have no “False” part. (; vs ,)
- Important: You MUST set a variable for the roll. Otherwise, calling `RandomInteger` four times will have you comparing different random numbers in each `If` statement.

Simulating rolling a biased die

```
aCount = 0; bCount = 0; cCount = 0; dCount = 0;
For[loopCount = 1, loopCount <= 20, loopCount++,
 roll=RandomReal[];
 If[ 0  <= roll < 1/2, Print["a"]; aCount++];
 If[1/2 <= roll < 3/4, Print["b"]; bCount++];
 If[3/4 <= roll < 7/8, Print["c"]; cCount++];
 If[7/8 <= roll <= 1 , Print["d"]; dCount++];]
distribution = {aCount, bCount, cCount, dCount}
```

- Sample output: (each on its own line)
  a, a, a, d, d, b, a, a, d, a, a, a, a, d, b, a, a, c, a, b $\{12, 3, 1, 4\}$
- These `If` statements all have no "False" part. (; vs ,)
- Important: You MUST set a variable for the roll. Otherwise, calling `RandomInteger` four times will have you comparing different random numbers in each `If` statement.
- If you are feeling fancy, you can use one `Which` command instead of four `If` commands.

Using Simulation to Calculate Area

Suppose you have a region whose area you don't know. You can approximate the area using a Monte Carlo simulation.

Using Simulation to Calculate Area

Suppose you have a region whose area you don't know. You can approximate the area using a Monte Carlo simulation.

Idea: Surround the region by a rectangle. Randomly chosen points in the rectangle will fall in the region with probability

(area of region)/(area of rectangle)

Using Simulation to Calculate Area

Suppose you have a region whose area you don't know. You can approximate the area using a Monte Carlo simulation.

Idea: Surround the region by a rectangle. Randomly chosen points in the rectangle will fall in the region with probability

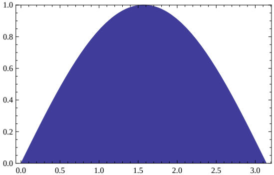
(area of region)/(area of rectangle)

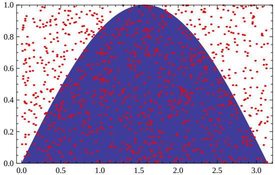
We can approximate this probability by calculating

(points falling in region)/(total points chosen).

Using Simulation to Calculate Area

Example. What is the area under the curve $\sin(x)$ from 0 to π?





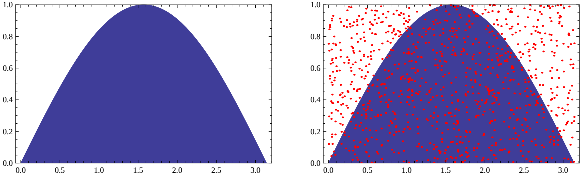
Randomly select 100 points from the rectangle $[0, \pi] \times [0, 1]$.

[*Choose a random real between* 0 *and* π *for the* x*-coordinate and a random real between* 0 *and* 1 *for the* y*-coordinate...*]

$$\text{Then, } \frac{\text{Area of region}}{\underline{\qquad\qquad}} \approx \frac{\text{Number of points in region}}{100}.$$

Using Simulation to Calculate Area

Example. What is the area under the curve $\sin(x)$ from 0 to π?



Randomly select 100 points from the rectangle $[0, \pi] \times [0, 1]$.

[*Choose a random real between* 0 *and* π *for the x-coordinate and a random real between* 0 *and* 1 *for the y-coordinate...*]

$$\text{Then, } \frac{\text{Area of region}}{\underline{\qquad\qquad}} \approx \frac{\text{Number of points in region}}{100}.$$

Here, 63 points fell in the region; we estimate the area to be ______.

Compare this to the actual value, $\int_{x=0}^{x=\pi} \sin(x)\, dx = [-\cos(x)]_{x=0}^{x=\pi} = 2$